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Competition and the risk of bank failure: Breaking with the representative borrower assumption

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Abstract
We examine the relation between intensity of competition in the loan market and risk of bank failure, in a model with adverse selection. As well established, the presence of the two opposite margin and risk-shifting effects creates conditions for nonmonotonicity: the conventional competition-fragility view may be challenged at high interest rates. These rates may however be too high to be compatible with oligopolistic equilibrium conditions. The challenging competition-stability view has been argued in terms of a representative borrower managing the profitability-safeness trade-off under moral hazard. However, the representative borrower assumption is not innocuous, playing down by construction the margin effect. The paper considers the adverse selection situation where that trade-off is managed by banks facing *heterogeneous* borrowers, and shows *analytically*, in the case of a trapezoidal distribution of idiosyncratic and systemic risk factors, that the conventional view is always valid.

1 | INTRODUCTION

The conventional wisdom concerning the relation between bank competition and bank stability is that more of the former undermines the latter. Erosion of market power, with the squeeze of profit margins, reduces banks' charter values, hence the penalty for failure, inducing riskier choices (Acharya, 1996; Keeley, 1990; Marcus, 1984). Boyd and De Nicolò (2005) challenged this view by taking into account the borrowers' standpoint. They claim that higher loan rates directly imply a higher risk of borrowers' bankruptcy and, by moral hazard, further create an incentive for borrowers to make riskier choices, an effect already highlighted by Stiglitz and Weiss (1981). With two opposite effects, the *margin* and the *risk-shifting* effects, one may wonder whether one of them tends to dominate the other, hence whether the relationship between intensity of competition and risk of bank failure is increasing, decreasing, or non-monotonic. Martinez-Miera and Repullo (2010) look for the relationship between the number of banks competing à la Cournot in the loan market and the probability of bank failure, under both idiosyncratic and systemic risk factors, and find that this relationship is U-shaped.[1]

Our purpose is to revisit this debate by proposing a model which, although close to the one adopted by Boyd and De Nicolò (2005) and by Martinez-Miera and Repullo (2010), is designed to take adverse selection, not moral hazard, as the source of the stabilizing risk-shifting effect. In Boyd and De Nicolò (2005), bank failure is identified with a *representative entrepreneur*'s default, since risks are *perfectly* correlated across the whole set of *identical* entrepreneurial projects, a context in which either *no* project fails or *all* projects fail together, with complementary probabilities. Then the risk of bank failure unambiguously decreases under more intense competition, which leads to a lower loan interest rate and, through the risk-shifting effect, to the choice of a less risky investment project by the representative entrepreneur. Still referring to a *representative entrepreneurial project*, with success probability decreasing in the loan interest rate, Martinez-Miera and Repullo (2010) concentrate on situations where project failures can be only partially correlated. In the limit case of perfectly correlated risks, they retrieve the Boyd and De Nicoló result and in the other limit case, of uncorrelated risks, any risk of bank default vanishes by portfolio diversification. In the intermediate cases where systemic and idiosyncratic risks coexist, the favorable effect on bank stability of stronger competition working through borrowers' decisions (the *risk-shifting effect*) combines with an unfavorable effect working through lower profit margins (the *margin effect*). Contrary to the competition-stability view, Martinez-Miera and Repullo show that the margin effect always dominates for a large enough number of banks (Proposition 2).[2] However, they also show by a numerical simulation that the risk-shifting effect dominates for a number of banks lower than a ceiling which increases with the degree of correlation, so that intensifying competition is favorable to bank stability, at least in highly concentrated loan markets and when borrowers' risks are highly correlated.

In this paper, we also assume Cournot competition in the loan market between a discrete number of identical banks, granting loans to a continuum of entrepreneurs. We abandon however the assumption that these entrepreneurs, while differing in terms of reservation utilities, have all access to the same technology and choose all the same project in response to a common loan interest rate, an assumption which results in an undiversified bank loan portfolio (except for a factor of idiosyncratic risk). By contrast, we assume that entrepreneurs, identical in terms of (zero) reservation utilities, have access to *heterogeneous* investment projects, each entrepreneur to a specific project. Bank loan portfolios are consequently diversified, with *aggregate* riskiness affected through adverse selection by the loan interest rate. We show that oligopolistic equilibrium conditions exclude the dominance of the risk-shifting effect over the margin effect in any observable situation: a dominating risk-shifting effect may occur only at loan interest rates that are larger than their highest enforceable value, hence essentially

[1]For further relevant references on the relation between bank competition and bank stability, see the surveys of the related theoretical and empirical literatures in Berger et al. (2009), Schaeck et al. (2009), Fungáčová and Weill (2013), Jiménez et al. (2013), and Jiang et al. (2017).

[2]They further emphasize the weakness of the risk-shifting effect, its modulus being constrained from above by the condition that the inverse loan demand should be decreasing.

nonobservable. To obtain an analytical result, we concentrate on the case where the risk factors and the safeness characteristic of the investment projects are uniformly distributed, resulting in a trapezoidal distribution of the combined risk, which mimicks the Gaussian bell. We show that the competition-fragility view always prevails in this case: an interest rate high enough to make the risk-shifting effect dominate the margin effect cannot be observed as an oligopolistic equilibrium outcome. We further show that the opposite result obtained under moral hazard, in particular when risk is purely systemic, is the direct consequence of the representative borrower assumption, independently of the choice of a specific distribution of risk factors.

The rest of the paper is organized as follows. In Section 2, we provide the model of bank competition for loans under adverse selection. In Section 3, we analyze with this model the relation between intensity of competition and risk of bank failure, concentrating on the case of the trapezoidal distribution of combined risk factors. This case allows one to obtain an analytical result comforting the conventional view. This result is then confronted with the one previously obtained in the literature under moral hazard. We propose an explanation for the difference. Section 4 concludes.

2 | A MODEL OF BANK COMPETITION FOR LOANS UNDER ADVERSE SELECTION

As already announced in Section 1, our modeling builds upon the seminal models of Boyd and De Nicolò (2005) and, principally, of Martinez-Miera and Repullo (2010), where banks compete in the loan market for the demand generated by a continuum of potential borrowers. In these models, the borrowers are investors having different reservation utilities to operate the same productive technology but being identical otherwise. This productive technology offers a trade-off between profitability and safeness, riskier projects having a larger return when they succeed. The trade-off is managed by the borrowers, in response to the loan rate (hence only indirectly by the rate-setting lenders), higher rates requiring projects with a higher success return but also with a lower probability of success.

By contrast, we consider a continuum of potential investors having the same (actually nil) reservation utility but being endowed with different technologies, each technology concerning a single project. Again, there is a trade-off between profitability and safeness, but this trade-off can now be managed by the lenders only. In our approach, *adverse selection*, not *moral hazard*, is the source of the increased riskiness, at higher loan rates, of the portfolios held by the banks.

In the three subsections of this section we deal successively with (i) entrepreneurial risk, and on its repercussion (ii) on the risk of loan portfolio default and (iii) on the equilibrium conditions of bank competition for loans.

2.1 | Entrepreneurial risk and demand for loans

We assume a continuum of unit mass of entrepreneurs, each entrepreneur having access to a single specific investment project. A project is characterized by an index x of profitability (its gross success return) and an index π of safeness. More profitable projects are riskier,[3] so that

[3]This assumption is the one made by Stiglitz and Weiss (1981) and contrasts with that, made by de Meza and Webb (1987), of equally profitable projects differing only by their probability of success. Here, we stick to the former, which allows one to compare our results with those of Boyd and De Nicolò (2005) and Martinez-Miera and Repullo (2010), obtained with it.

we assume that $x = \xi(\pi)$, where $\xi: [\underline{\pi}, \overline{\pi}] \to \mathbb{R}_+$ is a decreasing function, the index π being used in the following to identify a project and the entrepreneur endowed with that project. We further assume, as Stiglitz and Weiss (1981) do at first, that the set of projects $[\underline{\pi}, \overline{\pi}] \subset [0, 1]$ satisfies the mean preserving property, here $\pi\xi(\pi) = g \in (0, 1]$. Any project can be operated in period 0 at a fixed scale (normalized to one) so as to yield in period 1 the output $1 + x$ if it succeeds or the output $1 - \gamma$ (with a percentage loss $\gamma \in [0, 1]$) if it fails. Success depends upon the condition $\pi \geq Z(z_\pi)$, or $z_\pi \leq Z^{-1}(\pi)$, where $z_\pi \in \mathbb{R}$ is the realization of a random variable representing the risk impending on that project and $Z: \mathbb{R} \to [0, 1]$ is a continuous and increasing function making sense of the comparison between π and $Z(z_\pi)$.

Having no initial capital endowment, the entrepreneur must borrow one unit of capital, at date 0, to operate her project. After obtaining a loan from a bank at a positive *loan interest rate* R, she owes to that bank, at date 1, the principal 1 plus the interest R. However, by entrepreneur's *limited liability*, she will actually pay $1 + \min\{R, x\}$ if the project succeeds or $1 + \min\{R, -\gamma\} = 1 - \gamma$ if it fails. Consequently, if F is the cumulative distribution function (CDF) of random variable z_π, an entrepreneur has at date 0 the expected payoff

$$F \circ Z^{-1}(\pi)(\xi(\pi) - \min\{R, \xi(\pi)\}). \tag{1}$$

Taking the projects to be distributed according to the CDF $H: [\underline{\pi}, \overline{\pi}] \to [0, 1]$ and assuming that each entrepreneur will only borrow if her expected payoff is positive (if $\pi < g/R$), the demand for loans is $D(R) = H(g/R)$ for $g/R \in [\underline{\pi}, \overline{\pi}]$ (and 0 if $g/R < \underline{\pi}$, 1 if $g/R > \overline{\pi}$). An increase in R discourages the entrepreneurs endowed with less profitable and safer projects, so that (i) the demand for loans decreases and (ii) the average probability of success diminishes, since the withdrawn projects are the safest. Effect (ii) is the *risk-shifting effect*, already highlighted by Stiglitz and Weiss (1981), which works here through adverse selection and which we discuss in Subsection 2.2.

2.2 | The expected yield of loans and the risk of portfolio default

We assume that the random variable z_π ruling the outcome of investment project π combines additively an *idiosyncratic risk* factor s_π and a *systemic risk* factor S, common to all projects: $z_\pi = \zeta(1 - \rho)s_\pi + \zeta(\rho)S$. The variables s_π (for any project π) and S are all independent and identically distributed random variables with CDF Ψ. The parameter $\rho \in [0, 1]$ measures the correlation between the risks of the different projects and the function ζ is increasing and maps $[0, 1]$ onto itself. In the limit cases, the risks of two different projects π and π' are uncorrelated if $\rho = 0$ (since $z_\pi = s_\pi$ and $z_{\pi'} = s_{\pi'}$, s_π and $s_{\pi'}$ being independent random variables) and are perfectly correlated if $\rho = 1$ (since $z_\pi = z_{\pi'} = S$). In the following, we shall further take $Z \equiv \Psi$, whatever the adopted specification of the distribution Ψ. This modeling approach fits Vasicek (2002) representation of the equicorrelated normal distribution, used by Martinez-Miera and Repullo (2010), where all random variables (including z_π) have the same standard normal CDF Φ and ζ is the square root function. Another example of this representation of correlated risks, which will be exploited in Subsection 3.2, is given by random variables s_π and S uniformly distributed over $[0, 1]$, so that Ψ (but also ζ) is the identity function.[4] For $\rho \in (0, 1)$, the random

[4]To repeat, we will take in the example of next section $Z \equiv \Psi \equiv \zeta$ (but not F) as the identity function defined on $[0, 1]$, whereas Martinez-Miera and Repullo take $Z \equiv \Psi \equiv F$ as the standard normal CDF Φ (and ζ as the square root function).

variable z_π follows then a trapezoidal distribution, the density function of which has a graph which may be viewed as a stylized representation of the Gaussian bell (see Appendix A).

In this subsection we discuss the way the two risk factors determine the expected yield of loans and can ultimately lead to bank portfolio default. We show that the consequences of risk are quite different for its idiosyncratic and systemic factors. The former, not the latter, can be addressed by portfolio diversification and somewhat neutralized by the law of large numbers. We take this difference into account by introducing success probabilities conditional on some realization S of the systemic risk factor (when $\rho \in (0, 1)$). Since a project π succeeds if $Z^{-1}(\pi) \geq z_\pi = \zeta(1-\rho)s_\pi + \zeta(\rho)S$ (i.e., if $s_\pi \leq (Z^{-1}(\pi) - \zeta(\rho)S)/\zeta(1-\rho)$), it succeeds, conditionally on S, with probability

$$p_\pi(S) = \Psi\left(\frac{\Psi^{-1}(\pi) - \zeta(\rho)S}{\zeta(1-\rho)}\right), \tag{2}$$

taking into account the assumed coincidence of Z and Ψ.

As the index π is distributed according to the CDF H: $[\underline{\pi}, \bar{\pi}] \to [0, 1]$ (with density h), the probability of drawing a successful investment project among those that have actually been undertaken and conditional on the realization S, is the weighted mean

$$p(S, R) = \frac{1}{H(g/R)}\int_{\underline{\pi}}^{g/R} p_\pi(S)h(\pi)\,d\pi \text{ for } R \in [g/\bar{\pi}, g/\underline{\pi}]. \tag{3}$$

By the law of large numbers, this is also the *proportion of successful projects* in the investment portfolio *conditional on the realization S*. As p_π is a decreasing function, so is $p(\cdot,R)$. Moreover, as $p_\pi(S)$ is increasing in π, an increase in R, truncating from above the distribution of the investment projects, diminishes $p(S, R)$, so that $p(S, \cdot)$ is also a decreasing function, an expression of the risk-shifting effect. Notice that we are restricting R to the interval $[g/\bar{\pi}, g/\underline{\pi}]$, since the demand for loans vanishes altogether if $R > g/\underline{\pi}$, and the risk-shifting effect disappears if $R < g/\bar{\pi}$, as $p(S, R) = \int_{\underline{\pi}}^{\bar{\pi}} p_\pi(S)h(\pi)\,d\pi$ in that case.

We next analyze, for $R \in [g/\bar{\pi}, g/\underline{\pi}]$, how entrepreneurial risk may trigger bank portfolio default. The cost for the bank of the loan granted to an entrepreneur is the *deposit interest rate* r, which will be taken as exogenous and positive. So, the *conditional expected loan yield* (on the realization S of systemic risk) is

$$\begin{aligned} y(S, R) &= p(S, R)(1 + R) + (1 - p(S, R))(1 - \gamma) - (1 + r) \\ &= p(S, R)(R + \gamma) - (r + \gamma). \end{aligned} \tag{4}$$

The function $y(\cdot,R)$ is clearly decreasing. The *default threshold* $S^*(R)$ is the value of the systemic risk factor that triggers bank portfolio default, occurring for any $S > S^*(R)$, so that $1 - \Psi(S^*(R))$ is the probability of bank portfolio default. This default threshold is determined by the zero expected yield condition applying to the loan portfolio:

$$y(S^*, R) = p(S^*, R)(R + \gamma) - (r + \gamma) = 0. \tag{5}$$

Naturally, we are considering levels of the loan and deposit interest rates that ensure profitability but also riskiness of the financial intermediation activity, that is, $\sup_S y(S, R) > 0$ and $\inf_S y(S, R) < 0$, respectively, which entails a unique interior solution $S^*(R)$ to Equation (5). Of course, without systemic risk, if the investment project risks are uncorrelated ($\rho = 0$), the proportion of successful projects is still a decreasing function of R but it is not a random variable anymore: by (2) and (3), $p(\cdot,R)$ is a constant, the weighted mean of $p_\pi(\cdot) = \pi$. Equilibrium in the loan market excludes then bank failure (too high loan rates, leading with certainty to bank default, would never be chosen by profit-maximizing banks in the context of large diversified portfolios). Our analysis will accordingly be restricted to the case of correlated risks ($\rho > 0$).

To assess the effects on the default threshold of a variation in R, we use Equation (5) to compute by total differentiation the elasticity of $S^*(R)$:

$$\epsilon S^*(R) = \frac{\overbrace{\frac{R}{R+\gamma}}^{\text{margin effect}} + \overbrace{\epsilon_R p(S^*, R)}^{\text{risk-shifting effect}}}{-\epsilon_S p(S^*, R)}, \tag{6}$$

where $\epsilon_S p(S^*, R)$ and $\epsilon_R p(S^*, R)$ are the elasticities of the proportion of successful projects with respect to S and R, respectively. As the denominator on the right-hand side (RHS) of this equation is positive, the default threshold S^* depends negatively on R through the *risk-shifting effect*, since $\epsilon_R p(S^*, R) < 0$. However, it also depends positively on R through the *margin effect*: a higher R increases the loan yield $R - r$ when entrepreneurs succeed, providing a buffer to cover the loss per loan $r + \gamma$ when they fail. Intensity of competition, responsible for lower values of R, is thus seen to have through the margin effect an unfavorable influence, increasing the probability of portfolio default, but a favorable influence through the risk-shifting effect. Is the risk-shifting effect strong enough to undermine the conventional wisdom? We will address this question in Section 3.

2.3 | Bank competition for loans

We assume a number n of identical banks competing à la Cournot in a homogeneous loan market. In this context, each bank $j \in \{1, ..., n\}$ can be seen as setting a loan rate given by the inverse demand for loans: $R = D^{-1}\left(l_j + \sum_{j' \neq j} l_{j'}\right)$, where l_j is the demand for loans that bank j wishes to target and $l_{j'}$ the demand for loans conjectured as the target of bank j'. The target l_j is chosen so as to maximize the ensuing expected profit $l_j Y(R) = l_j Y\left(D^{-1}\left(l_j + \sum_{j' \neq j} l_{j'}\right)\right)$, where $Y(R)$ is the (unconditional) expected loan yield. In a symmetric equilibrium, the first-order condition for that maximization is

$$\frac{1}{\epsilon Y(R)} = \frac{1/n}{-\epsilon D(R)}, \tag{7}$$

where $\epsilon D(R)$ and $\epsilon Y(R)$ are the elasticities of the demand for loans and of the expected loan yield, respectively.

When determining the *unconditional* expectation of the loan yield $Y(R)$, we have to take the bank *limited liability* into account, so that Equation (4), defining the expected loan yield *conditional* on the realization S of systemic risk, applies for $S \leq S^*$ only, this yield being otherwise zero. The loan yield that bank j expects to obtain is consequently

$$\begin{aligned} Y(R) &\equiv \int_{-\infty}^{S^*(R)} y(S, R)\psi(S)\, dS \\ &= \Psi(S^*(R))\left(\underbrace{\left(\frac{1}{\Psi(S^*(R))}\int_{-\infty}^{S^*(R)} p(S, R)\psi(S)\, dS\right)}_{P(R)}(R + \gamma) - (r + \gamma)\right), \end{aligned} \tag{8}$$

where $\psi(S)$ is the probability density of S and $P(R)$ is the *expected proportion of successful projects*. We can then compute the elasticity $\epsilon Y(R)$ of the expected loan yield[5] and use the result in the profit maximization condition (7). We thus get the following expression for the *Lerner index of market power*, namely, the relative markup of the expected yield per loan $P(R)(R+\gamma)$ over its marginal cost $r+\gamma$:

$$\frac{P(R)(R+\gamma)-(r+\gamma)}{P(R)(R+\gamma)} = \frac{R/(R+\gamma)+\epsilon P(R)}{n(-\epsilon D(R)) - \epsilon\Psi(S^*(R))\epsilon S^*(R)}. \tag{9}$$

This equation determines the equilibrium value of the loan interest rate R as a function of the number n of competing banks. The first term in the denominator of its RHS is the reciprocal of the usual Cournot's degree of monopoly $(1/n)/(-\epsilon D(R))$. The second term in the denominator expresses the effect on market power induced by bank limited liability, strengthening if S^* is increasing (the competition-fragility view), weakening if S^* is decreasing (the competition-stability view). The two terms in the numerator express the two effects of a change in the loan interest rate which have already been highlighted, namely, the margin effect and the risk-shifting effect. The former is always positive and the latter, which now applies to the (unconditional) expected proportion of successful projects, not to the proportion *conditional on* $S^*(R)$ as in Equation (6), is negative if S^* is increasing but might be positive if S^* is decreasing.[6]

3 | INTENSITY OF COMPETITION AND RISK OF BANK FAILURE

The question we now want to address concerns the sense of the response of the probability $1-\Psi(S^*)$ of bank portfolio default to a change in the intensity of competition, as usual indexed by the number $n \in \mathbb{N}\backslash\{0,1\}$ of competing banks,[7] through the resulting variation of the loan interest rate R. Clearly, if the default threshold S^* is decreasing in R, a higher intensity of competition, depressing the loan interest rate, leads to a lower probability of bank failure, thus vindicating the competition-stability view. The relevant question is however not that of the possibility of getting a decreasing S^* for some set of values of R. It is the possibility of getting such property for a set of admissible values of R, *observable as a Cournot equilibrium outcome*. In the following, we briefly address this question in general terms and then examine the case of uniform distributions of characteristics and risk which, contrary to the case of Gaussian risk, allows one to establish an analytical result. We finally compare this result with those obtained in the literature under moral hazard.

[5]We obtain

$$\epsilon Y(R) = \epsilon\Psi(S^*(R))\epsilon S^*(R) + \frac{P(R)(R+\gamma)}{P(R)(R+\gamma)-(r+\gamma)}\left(\frac{R}{R+\gamma}+\epsilon P(R)\right).$$

[6]Indeed, $p(S,R)$ is decreasing in R but is averaged, for S^* responding negatively to a higher R, over a smaller set, reduced to its highest values (see Equation 8).

[7]The intensity of competition cannot of course be reduced to market structure. It also depends upon the more or less aggressive competitors' conduct, upon the extent of substitutability between the products or upon the easiness to enter the market. In the present context, which allows comparisons with the relevant literature, the number of banks appears however as the appropriate index of competitive intensity,

3.1 | Observable risk-decreasing competition: a preliminary discussion

Negativity of $\epsilon S^*(R)$ means, by Equation (6), that the risk-shifting effect conditional on $S^*(R)$, namely, $\epsilon_R p(S^*, R)$ dominates, at least locally, the margin effect $R/(R+\gamma)$, that is, that $R/(R+\gamma) < -\epsilon_R p(S^*, R)$. However, referring to Equation (9), we see that, since the Lerner index must be positive at any oligopolistic equilibrium, $\epsilon S^*(R) < 0$ implies that the risk-shifting effect is dominated by the margin effect *on average* (for values of S smaller than S^*), that is, that $-\epsilon P(R) < R/(R+\gamma)$. We thus end up with the following condition for the competition-stability view to hold in oligopolistic equilibria.

Condition 1. A necessary condition for S^* to be decreasing at an equilibrium value R^* of the loan interest rate is that

$$-\epsilon P(R^*) < \frac{R^*}{R^* + \gamma} < -\epsilon_R p(S^*, R^*). \tag{10}$$

Let us consider the implications of this condition. Referring to Equation (8), which gives the definition of P as the average value of the function $p(\cdot,R)$ on the interval $(-\infty, S^*(R)]$, we can compute the elasticity (in absolute value) of this function:

$$-\epsilon P(R) = \epsilon S^*(R) \underbrace{\epsilon\Psi(S^*)\left(1 - \frac{p(S^*(R),R)}{P(R)}\right)}_{>0} + \frac{\int_{-\infty}^{S^*(R)} -\epsilon_R p(S,R) p(S,R) \psi(S)\, dS}{\int_{-\infty}^{S^*(R)} p(S,R)\psi(S)\, dS}. \tag{11}$$

The last term on the RHS is the average value of the elasticity $-\epsilon_R p(S, R)$, which is a measure of the risk-shifting effect. If $-\epsilon_R p(\cdot,R)$ is an increasing function (if the risk-shifting effect is strengthened by any increase in the systemic risk), its average value is smaller than $-\epsilon_R p(S^*, R)$, so that the condition $-\epsilon P(R) < -\epsilon_R p(S^*, R)$ is actually *implied* by $\epsilon S^*(R) < 0$, as stated in the following condition.

Condition 2. For S^* decreasing at an equilibrium value R^*, the condition that $-\epsilon_R p(\cdot,R^*)$ be increasing over $(-\infty, S^*(R^*)]$ is sufficient for $-\epsilon P(R^*)$ to be smaller than $-\epsilon_R p(S^*, R^*)$.

Thus, although neither necessary nor alone sufficient, the condition of a risk-shifting effect strengthening with systemic risk is at least favorable to the validation of the competition-stability view.

3.2 | The case of uniform risk and characteristic distributions

In this subsection, the index π of safeness of investment projects as well as the two risk factors s and S are assumed to be uniformly distributed over the interval $[0, 1]$, so that the functions H and Ψ are both taken as the identity function. However, the distribution of the variable $z_\pi = (1-\rho)s_\pi + \rho S$, a convex combination of the two uniformly distributed risk factors, is trapezoidal for $\rho \in (0, 1)$ (see Appendix A). As to the demand for loans, it is now $D(R) = \min\{g/R, 1\}$.

3.2.1 | Bank default for observable loan interest rates under pure systemic risk

Let us first consider the most favorable case for the competition-stability view, that of pure systemic risk ($\rho = 1$) which, in Boyd and De Nicolò (2005) and Martinez-Miera and Repullo (2010), *always* fits that view. The proportion of successful projects conditional on S is then, by Equations (2) and (3) and taking $\rho = 1$ (which entails $F(z) \equiv z = S$),

$$p(S, R) = \max\{1 - SR/g, 0\}, \tag{12}$$

with average

$$P(R) = 1 - S^*(R, r)R/2g, \text{ where } S^*(R) = g(1 - r/R)/(R + \gamma). \tag{13}$$

The elasticity $-\epsilon_R p(S, R) = (SR/g)/(1 - SR/g)$ is increasing in S as long as $p(S, R) > 0$, verifying Condition 2 (with a decreasing S^*), which is a good first step to vindicate the competition-stability view.

Is this property sufficient? We show that it is not, by taking equilibrium conditions into account. Indeed, we prove that the oligopoly equilibrium loan interest rate $R^*(n)$ for any number $n \geq 2$ of competing banks is smaller than the value $\hat{R} = \arg\max S^*(R)$, and hence belongs to the interval $(r, \widehat{R})$ over which this (strictly concave) function is increasing.

Proposition 1. *Let π and $z_\pi = S$ be both uniformly distributed over* $[0, 1]$. *Then the default threshold S^*, although a unimodal function of the loan interest R, is increasing for any $R \leq R^*(2)$, the highest Cournot equilibrium rate.*

Proof. The unique maximum of S^* is attained at $\widehat{R} = r(1 + \sqrt{1 + \gamma/r})$. Using the Lerner index equation (9), it is straightforward to compute the equilibrium loan interest rate $R^*(n)$. We first compute

$$\begin{aligned} &P(R) = \frac{R + r + 2\gamma}{2(R + \gamma)}, - \epsilon P(R) = \frac{r + \gamma}{R + r + 2\gamma}\frac{R}{R + \gamma}, \\ &\text{and } \epsilon S^*(R) = \frac{r}{R - r} - \frac{R}{R + \gamma}, \end{aligned} \tag{14}$$

then obtaining from (9)

$$\begin{aligned} R^*(n) = {} & r\frac{n(1 - \gamma/r) + 2 + \gamma/r}{2n} \\ & + r\sqrt{\left(\frac{n(1 - \gamma/r) + 2 + \gamma/r}{2n}\right)^2 + \frac{n + 1}{n}\gamma/r}, \end{aligned} \tag{15}$$

which is clearly a decreasing function, as expected. The duopoly equilibrium rate $R^*(2)$ is consequently the highest oligopoly equilibrium rate. It is easy to check that $R^*(2) < \widehat{R} < R^*(1)$. □

Thus, decreasing the loan interest rate from $R^*(2)$ through an intensification of competition (a higher n) can only increase the risk of bank failure since S^* is an increasing function for $R \in (\max\{r, g\}, \widehat{R})$. This situation is illustrated in Figure 1, which represents the function S^* (with $r = 0.02$ and $\gamma = 0.2$). The abscissa of the vertical solid line indicates the maximum Cournot equilibrium loan rate, $R^*(2)$. Intensifying competition through an increase in the number of banks translates into successive shifts of the vertical line to the left, as represented by the dashed lines (corresponding to $n = 3, 4, 5$ and 6, successively). With each such shift is associated a lower and lower value of S^*, hence a higher and higher probability $1 - S^*$ of bank default.

Movements along the graph of S^* to the right of $R^*(2)$ are not observable in the presence of bank competition, which requires at least two competitors. It is true that, for $R \in (\widehat{R}, R^*(1))$, the risk of bank default diminishes as the loan interest rate decreases from its monopoly value $R^*(1)$. This can however hardly be seen as a local validation of the competition-stability view, since competition concerns a discrete number of competitors, requiring at least two of them. We will avoid the semantic issue that consists in arguing that the intensity of competition is also raised when switching to duopoly from monopoly, where it is zero. Anyway, we can check by a simple computation that $S^*(R^*(2)) \leq S^*(R^*(1))$ (as in Figure 1) provided $\gamma/r \geq 5$, which seems reasonable to assume ($\gamma = 0.45$ and $r = 0$ in the main numerical simulation of Martinez-Miera & Repullo, 2010). The competition-stability view is then rejected again.

In spite of an inverse U-shaped graph of the threshold S^* of bank failure, in spite of the fulfillment of Condition 2, the conditions for an oligopolistic equilibrium with profit-maximizing banks keep on validating the conventional competition-fragility view when the risk is reduced to its systemic component. This result is the exact opposite of the one in Boyd and De Nicolò (2005) and Martinez-Miera and Repullo (2010) when the risk is purely systemic. The reason for this discrepancy will be discussed in Subsection 3.3. Before that discussion, we shall however extend our analysis, in Subsubsection 3.2.2, to the case where systemic and idiosyncratic risks coexist.

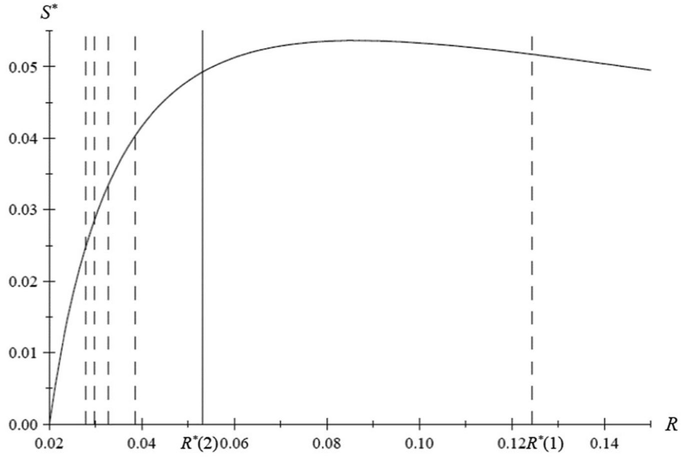

FIGURE 1 Bank default under pure systemic risk

3.2.2 | Bank default for observable loan interest rates under systemic and idiosyncratic risks

By Equations (2) and (3), with $\Psi \equiv \zeta \equiv H$ taken as the identity function, the probability of success of the investment project π conditional on the realization S of systemic risk is, for $0 < \rho < 1$ and $g/R \leq 1$,

$$\begin{aligned} &p_{\pi}(S) = \frac{\pi - \rho S}{1-\rho} \quad \text{if } \pi \in [\rho S, 1-\rho+\rho S], \\ &p_{\pi}(S) = 0 \quad \text{if } \pi \in [0, \rho S], \text{ and } p_{\pi}(S) = 1 \text{ if } \pi \in [1-\rho+\rho S, 1], \end{aligned} \tag{16}$$

leading to the proportion of successful projects

$$\begin{aligned} p(S,R) &= \frac{1}{g/R}\left(\int_{\rho S}^{\min\{g/R, 1-\rho+\rho S\}} \frac{\pi-\rho S}{1-\rho} d\pi + \max\{g/R - (1-\rho+\rho S), 0\}\right) \\ &= \left\{\begin{matrix} 1 - \frac{1-\rho+2\rho S}{2g/R} & \text{if} \quad 1-\rho+\rho S \leq g/R, \\ \frac{(g/R - \rho S)^2}{2(1-\rho)g/R} & \text{if} \quad 1-\rho+\rho S \geq g/R. \end{matrix}\right) \end{aligned} \tag{17}$$

By (5), this leads in turn to the bank default threshold

$$S^*(R) = \left\{\begin{matrix} \frac{1}{\rho}\left(\frac{g}{R}\frac{R-r}{R+\gamma} - \frac{1-\rho}{2}\right) & \text{if} \quad r \leq R \leq \bar{R}(\rho), \\ \frac{g}{\rho R}\left(1 - \sqrt{2(1-\rho)\frac{r+\gamma}{g}\frac{R}{R+\gamma}}\right) & \text{if} \quad R \geq \bar{R}(\rho), \end{matrix}\right) \tag{18}$$

where

$$\overline{R}(\rho) \equiv \sqrt{(\gamma/2)^2 + 2g(r+\gamma)/(1-\rho)} - \gamma/2. \tag{19}$$

Assuming profitabiliy and riskiness of the loan portfolio, as we do, means restricting parameter values so as to ensure that $S^*(R)$ as defined belongs to the interval $(0, 1)$. Recall that, when $\rho = 1$, the function S^* has the unique maximum

$$\widehat{R} = r(1 + \sqrt{1+\gamma/r}) = \arg\max_R((1 - r/R)/(R+\gamma)). \tag{20}$$

This is also the maximum of function S^* in the specification introduced by (18) for $R \in [r, \overline{R}(\rho)]$. Thus, if $\rho < 1$ and as S^* is decreasing for $R > \overline{R}(\rho)$, the function S^* is unimodal, increasing in the interval $(r, \min\{\overline{R}(\rho), \widehat{R}\})$ and decreasing for $R > \min\{\overline{R}(\rho), \widehat{R}\}$. Whatever the configuration of parameter values (compatible with profitability and riskiness of the loan portfolio), the risk of bank default eventually increases with the loan interest rate, apparently vindicating the competition-stability view, at least when the loan interest rate is high. But are loan interest rates higher than $\min\{\overline{R}(\rho), \widehat{R}\}$ observable at some equilibrium?

To answer to this question, we must distinguish two cases according to the relative position of $\overline{R}(\rho)$ and $\widehat{R}$. The first case, where the maximum of S^* is attained at $\overline{R}(\rho)$, smaller than $\widehat{R}$, results from a high-cost γ of borrower's default relative to lending cost r. More precisely, it corresponds to parameter values such that

$$\frac{\sqrt{1+\gamma/r}}{(1+\sqrt{1+\gamma/r})^2} < \frac{1-\rho}{2g/r}, \tag{21}$$

namely, a large value of γ/r, together with a low correlation ρ between borrowers' risks and a small expected return g/r from banks' intermediation. We can show that the expected loan yield is then negative for values of the loan interest rate higher than $\overline{R}(\rho)$, excluding the observability of interest rates belonging to the interval where S^* is decreasing.

The second case, where the maximum of S^* is attained at $\widehat{R}$, smaller than $\overline{R}(\rho)$, results from low values of γ/r or, more precisely, from a restriction of the parameter values opposite to condition (21). This is the case which extends to the limit situation of pure systemic risk, covered by Proposition 1. We can show that Cournot equilibrium values of the loan interest rate are, also in this case, necessarily smaller than $\widehat{R}$, validating the competition-fragility view. The following proposition generalizes the result stated in Proposition 1.

Proposition 2. *Let π, s_π, and S be all uniformly distributed over $[0, 1]$ and z_π be a convex combination of s_π and S with positive weight on S. Then the default threshold S^*, a unimodal function of the loan interest R, is increasing for any $R \leq R^*(2)$, the highest Cournot equilibrium rate.*

Proof. See Appendix B. □

The competition-stability view is consequently rejected, whatever the parameter values ρ, g, γ, and r, if we take oligopolistic equilibrium conditions into account. Any increase in the intensity of competition due to bank creation augments the risk of bank default through the ensuing decrease in the equilibrium loan rate.

3.3 | Comparison with the corresponding moral hazard representative investor case

Our results are substantially different from those of Boyd and De Nicolò (2005) and Martinez-Miera and Repullo (2010). They are even their exact opposite when the risk is purely systemic. Why? As already recalled, all investors are assumed in these two papers to have access to the same technology, an assumption which would translate in our framework into a degenerate distribution of the safeness characteristic, with $\underline{\pi} = \overline{\pi} = \widehat{\pi}$. This would result for $\rho = 1$ and $R \leq g/\widehat{\pi}$ in a proportion of successful projects

$$p(S, R) = 1 \text{ if } S \leq \hat{\pi} \text{ and } p(S, R) = 0 \quad \text{if } S > \hat{\pi}. \tag{22}$$

As a consequence, $S^* = \widehat{\pi}$ independently of R and the margin effect implied by the zero expected yield condition (5) would be lost. As to the risk-shifting effect, it is introduced in these two papers by the possibility for the representative investor to choose π so as to maximize her expected payoff. In the standard normal specification, $F \equiv Z \equiv \Psi \equiv \Phi$, so that the probability of success of project π is $F \circ Z^{-1}(\pi) = \pi$ and, by (1), this choice gives

$$\hat{\pi}(R) = \arg\max_{\pi} (\pi(\xi(\pi) - R)). \tag{23}$$

If a solution exists,[8] $\hat{\pi}$ is also decreasing, entailing the existence of a risk-shifting effect, here ascribable to moral hazard. We thus end up with a function $S^*(R) = \hat{\pi}(R)$ which is always decreasing because the negative risk-shifting effect is not counteracted by the positive margin effect.

What about the case $\rho \in (0, 1)$ where idiosyncratic and systemic risk factors coexist? Starting with the case of uniform risk distributions and by Equations (2) and (3), with Ψ and ζ taken as the identity function, the proportion of successful projects conditional on S (which coincides with the probability of success of the representative project since $[\underline{\pi}, \overline{\pi}]$ degenerates into $\hat{\pi}$) is

$$p(S, R) = p_{\hat{\pi}(R)}(S) = \frac{\hat{\pi}(R) - \rho S}{1 - \rho} \quad \text{for } S \in \left[\frac{\hat{\pi}(R) - (1 - \rho)}{\rho}, \frac{\hat{\pi}(R)}{\rho}\right], \tag{24}$$

if $0 < (\hat{\pi}(R) - (1 - \rho))/\rho < \hat{\pi}(R)/\rho < 1$. The proportion $p(S, R)$ of successful projects takes now all the values between 0 and 1, in particular, according to Equation (5), the value $(r + \gamma)/(R + \gamma)$ so as to determine $S^*(R)$. This opens the way to the margin effect but, as long as ρ remains close to 1, the width of the interval given by Equation (24) for $p(\cdot,R)$ to vary and within which the margin effect operates is small. In the complementary interval $[0, (\hat{\pi}(R) - (1 - \rho))/\rho] \cup [\hat{\pi}(R)/\rho, 1]$, $p(S, R)$ is either 1 or 0. Hence, the influence of R on S^* through the margin effect is confined to a narrow range and accordingly dominated by the impact of R on $\hat{\pi}(R)$, which is able to shift the interval of variation of $p(\cdot,R)$. In other words, the margin effect is bridled by construction relative to a risk-shifting effect entirely ascribable to moral hazard.

We can easily extend this conclusion to other risk distributions, in particular the Gaussian one used by Martinez-Miera and Repullo (2010), with $\Psi \equiv \Phi$ the standard normal CDF and ζ the square root function. Indeed, if $p(S, R)$ varies now with S along the whole real line, levels of ρ close to 1 lead to high absolute values of $(\Phi^{-1}(\hat{\pi}(R)) - \sqrt{\rho}S)/\sqrt{1 - \rho}$, belonging to the queues of the distribution, so that $\Phi((\Phi^{-1}(\hat{\pi}(R)) - \sqrt{\rho}S)/\sqrt{1 - \rho})$ is then largely inelastic with respect to S. Again, the adjustment through S and then Φ to variations in $(r + \gamma)/(R + \gamma)$ according to Equation (5)—the margin effect—is significant only within a narrow range when ρ is close to 1.

From the preceding discussion it becomes obvious that it is the representative borrower assumption that is responsible for the dominance of the risk-shifting effect when the risk is predominantly systemic. Indeed, as just shown, it seriously holds down the margin effect, eliminating it in the limit case $\rho = 1$, independently of the choice of the probability distribution for the two risk factors.

4 | CONCLUSION

We analyzed the relation between intensity of competition in the loan market and risk of bank failure using a model which considers adverse selection as the source of the increased riskiness, at higher loan interest rates, of the portfolios held by the banks. This risk-shifting effect,

[8] Existence is warranted if ξ is not too convex $(-\xi''(\pi)\pi/\xi'(\pi) < 2)$. Notice that the mean preserving condition $\xi(\pi)\pi = g$ does not apply anymore in this context.

working through the borrowers' channel, counteracts the margin effect which works through the lenders' channel and on which is based the conventional view that more intense competition is detrimental to bank stability. The conflict between the two effects accounts for nonmonotonicity of the relation, with a dominating margin effect at low interest rates (when bank competition is intense) and a dominating risk-shifting effect at high interest rates (when bank competition is weak).

However, the whole of this relation is not necessarily observable: our first contribution is to emphasize the fact that a decrease in the loan interest rate may well diminish the probability of bank default but only at high levels of that rate, actually too high to be compatible with an oligopolistic equilibrium, as they would create an incentive for banks to spontaneously deviate to lower rates. Our second contribution is to question the robustness of the results obtained under the representative borrower assumption. This assumption is not only strong, it directly commands the outcome of a dominating risk-shifting effect by severely confining the margin effect when risks are highly correlated, even eliminating it altogether in the limit case of pure systemic risk. By contrast, our analysis consolidates the conventional competition-fragility view, not by denying the importance of the risk-shifting effect as loan interest rates become high, but by bringing to the fore oligopolistic equilibrium conditions under which those high rates can in fact not be enforced.

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REFERENCES

Acharya, S. (1996). Charter value, minimum bank capital requirement and deposit insurance pricing in equilibrium. *Journal of Banking & Finance*, *20*, 351–375.

Berger, A. N., Klapper, L. F., & Turk-Ariss, R. (2009). Bank competition and financial stability. *Journal of Financial Services Research*, *35*, 99–118.

Boyd, J. H., & De Nicolò, G. (2005). The theory of bank risk taking and competition revisited. *Journal of Finance*, *60*, 1329–1343.

Fungáčová, Z., & Weill, L. (2013). Does competition influence bank failures? Evidence from Russia. *Economics of Transition*, *21*, 301–322.

Jiang, L., Levine, R., & Lin, C. (2017). *Does competition affect bank risk*? NBER Working Paper No. 23080.

Jiménez, G., Lopez, J. A., & Saurina, J. (2013). How does competition affect bank risk-taking? *Journal of Financial Stability*, *9*, 185–195.

Keeley, M. C. (1990). Deposit insurance, risk, and market power in banking. *American Economic Review*, *80*, 1183–1200.

Marcus, A. J. (1984). Deregulation and bank financial policy. *Journal of Banking & Finance*, *8*, 557–565.

Martinez-Miera, D., & Repullo, R. (2010). Does competition reduce the risk of bank failure? *Review of Financial Studies*, *23*, 3638–3664.

de Meza, D., & Webb, D. C. (1987). Too much investment: A problem of asymmetric information *Quarterly Journal of Economics*, *102*, 281–292.

Schaeck, K., Cihak, M., & Wolfe, S. (2009). Are competitive banking systems more stable? *Journal of Money, Credit and Banking*, *41*, 711–734.

Stiglitz, J. E., & Weiss, A. (1981). Credit rationing in markets with imperfect information. *American Economic Review*, *71*, 393–410.

Vasicek, O. A. (2002). The distribution of loan portfolio value. *Risk*, *15*, 160–62.

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APPENDIX A: THE TRAPEZOIDAL DISTRIBUTION

For $s_\pi \in [0, 1]$ and $S \in [0, 1]$, the random variable z_π is the sum of two independent random variables $(1-\rho)s_\pi$ and ρS, uniformly distributed over $[0, 1-\rho]$ and $[0, \rho]$, respectively. All the z_π's follow the same trapezoidal distribution, with CDF[9]

$$F(z) = \begin{cases} \frac{z^2}{2\rho(1-\rho)} & \text{if } 0 \le z \le \min(\rho, 1-\rho), \\ \frac{z - \min(\rho, 1-\rho)/2}{\max(\rho, 1-\rho)} & \text{if } \min(\rho, 1-\rho) \le z \le \max(\rho, 1-\rho), \\ 1 - \frac{(1-z)^2}{2\rho(1-\rho)} & \text{if } \max(\rho, 1-\rho) \le z \le 1 \end{cases} \tag{A1}$$

and probability density function (PDF)

$$f(z) = \begin{cases} \frac{z}{\rho(1-\rho)} & \text{if } 0 \le z \le \min(\rho, 1-\rho), \\ \frac{1}{\max(\rho, 1-\rho)} & \text{if } \min(\rho, 1-\rho) \le z \le \max(\rho, 1-\rho), \\ \frac{1-z}{\rho(1-\rho)} & \text{if } \max(\rho, 1-\rho) \le z \le 1. \end{cases} \tag{A2}$$

The graph of f is a trapezoid, degenerating into a triangle for $\rho = 1/2$ and into a horizontal line segment for $\rho = 0$ and 1. The PDF f is continuous, so that the CDF F is differentiable.

APPENDIX B: PROOF OF PROPOSITION 2

We consider successively the two cases characterized by the relative positions of $\overline{R}(\rho)$ and $\widehat{R}$, as defined by Equations (19) and (20) and as determined by the parameter values γ/r, g/r, and ρ.

- $\overline{R}(\rho) < \widehat{R}$

In this case, resulting from condition (21), with a high relative value of γ/r, the function S^*, defined by (18), is maximized at $\overline{R}(\rho)$ where it attains the value

$$S^*(\overline{R}(\rho)) = \frac{1}{\rho}\left(\frac{g}{\overline{R}(\rho)}\frac{\overline{R}(\rho) - r}{\overline{R}(\rho) + \gamma} - \frac{1-\rho}{2}\right). \tag{B1}$$

[9]To check the expression for the CDF $F(z)$, as all the points in the space $[0, \rho] \times [0, 1-\rho]$ are equiprobable, just compute the areas of the regions defined by $(1-\rho)s + \rho S \le z$ relative to the total area $\rho(1-\rho)$ of the rectangle.

By Equations (8) and (17), the expected proportion of successful projects at $\overline{R}(\rho)$ is

$$P(\bar{R}(\rho)) = \frac{1}{S^*(\bar{R}(\rho))}\int_0^{S^*(\bar{R}(\rho))}\left(1 - \frac{1-\rho+2\rho S}{2g\,/\,\bar{R}(\rho)}\right)dS \\ = 1 - \frac{1-\rho+\rho S^*(\bar{R}(\rho))}{2g\,/\,\bar{R}(\rho)}. \tag{B2}$$

By using these two equations and Equation (19), we obtain

$$P(\overline{R}(\rho)) = 1/2. \tag{B3}$$

For the expected loan yield $Y(\overline{R}(\rho)) = S^*(\overline{R}(\rho))[P(\overline{R}(\rho))(\overline{R}(\rho) + \gamma) - (r + \gamma)]$ to be positive, we then have (using again Equation 19)

$$\frac{1-\rho}{2g/r} < \frac{1}{2(2+\gamma/r)}. \tag{B4}$$

This inequality is incompatible with the restriction (21), so that the expected loan yield is always negative at $\overline{R}(\rho)$ in this case. Moreover, $P(R)(R+\gamma)$ is decreasing for $R > \overline{R}(\rho)$ (for $\epsilon S^*(R) < 0$), since

$$\epsilon(P(R)(R+\gamma)) = \epsilon P(R) + \frac{R}{R+\gamma} < 0 \text{ if } P(R)(R+\gamma) < r+\gamma, \tag{B5}$$

by the Lerner index equation (9). Hence, the expected loan yield cannot become positive again, once $R > \overline{R}(\rho)$. As a consequence, loan interest rates higher than $\overline{R}(\rho)$ cannot be observed in equilibrium and the bank default threshold $S^*(R)$ is increasing for any observable R, smaller than $\overline{R}(\rho)$.

- $\widehat{R} \le \overline{R}(\rho)$

In this case, resulting from the opposite of condition (21), namely,

$$\frac{1-\rho}{2g/r} \le \frac{\sqrt{1+\gamma/r}}{(1+\sqrt{1+\gamma/r})^2}, \tag{B6}$$

with a low relative value of γ/r, the function S^* is maximized at $\widehat{R}$. We then have, for $R < \overline{R}(\rho)$ and by Equations (17) and (18),

$$p(S,R) = 1 - \frac{1-\rho+2\rho S}{2g/R} \quad \text{and} \quad S^*(R) = \frac{1}{\rho}\left(\frac{g}{R}\frac{R-r}{R+\gamma} - \frac{1-\rho}{2}\right). \tag{B7}$$

Also, by Equation (8),

$$P(R) = 1 - \frac{1-\rho+\rho S^*(R)}{2g/R} = 1 - \frac{1}{2}\frac{R-r}{R+\gamma} - \frac{1-\rho}{4g/R}. \tag{B8}$$

Again, for R to be observable as an equilibrium outcome, a positive expected loan yield $Y(R) = S^*(R)[P(R)(R+\gamma) - (r+\gamma)]$ is required, a condition which translates into

$$\frac{1-\rho}{2g/r} < \frac{1}{R/r}\frac{R/r-1}{R/r+\gamma/r}. \tag{B9}$$

The RHS of this equation is maximized in R/r at $\widehat{R}/r = 1 + \sqrt{1 + \gamma/r}$, so that the loan yield positivity condition implies

$$\frac{1-\rho}{2g/r} < \frac{1}{(1 + \sqrt{1+\gamma/r})^2}, \tag{B10}$$

an inequality which reinforces our primary restriction (B6) of the parameter values.

The Lerner index equation (9), applying at equilibrium, can be rewritten as

$$\begin{aligned} n &= \frac{P(R) + P^{'}(R)(R+\gamma)}{P(R)(R+\gamma) - (r+\gamma)} R + \epsilon S^*(R) \\ &= 1 + \frac{1 - \frac{1-\rho}{2g/r}(R/r)^2}{\left(1 - \frac{1-\rho}{2g/r}\gamma/r\right)R/r - 1 - \frac{1-\rho}{2g/r}(R/r)^2} \\ &\quad + \frac{1 - \frac{R/r-1}{R/r+\gamma/r}R/r}{\left(1 - \frac{1-\rho}{2g/r}\gamma/r\right)R/r - 1 - \frac{1-\rho}{2g/r}(R/r)^2}. \end{aligned} \tag{B11}$$

The sign of the derivative with respect to R/r of the first variable term on the RHS of this equation is given by a quadratic, which is negative for $R/r \in (\widehat{R}/r, \overline{R}(\rho)/r)$ under condition (B10). The second variable term, expressing the elasticity $\epsilon S^*(R)$, is also decreasing at least in a neighborhood of $\widehat{R}/r$, a point where it switches from positive to negative. We thus obtain, as expected, a decreasing relation $n = n^*(R/r)$ between the number of banks and the (normalized) loan interest rate at equilibrium. The largest possible value n is necessarily lower, in the relevant interval $(\widehat{R}/r, \overline{R}(\rho)/r)$ where S^* is decreasing ($\epsilon S^*(R) < 0$), than the value obtained from Equation (B11) for $R/r = \widehat{R}/r = 1 + \sqrt{1+\gamma/r}$, namely,

$$n^*(\widehat{R}/r) = 1 + \frac{1}{\sqrt{1+\gamma/r}} \leq 2. \tag{B12}$$

This completes the proof.[10] □

[10] As in the case of pure systemic risk, we may evoke the possibility of considering the switch from monopoly to duopoly as an intensification of bank competition in the loan market. Again, with a high enough value of the ratio γ/r, hence a value of $n^*(\widehat{R}/r)$ close enough to 1, we shall obtain $S^*(R^*(2)) < S^*(R^*(1))$, fully rejecting the competition-stability view.