

## CHAPTER 11

# Empirical estimation of drug treatment costs in Portugal

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### Introduction

Illicit drugs have a significant impact on users and society, and the burden of health-related problems resulting from their use is enormous. The approach to drug policy varies widely across countries and enforcement still consumes the bulk of public resources, even in countries with an element of decriminalisation such as the Netherlands (Rigter, 2006) <sup>(1)</sup>. However, it is now apparent that drug abuse and addiction treatment is much less expensive than its alternatives (such as imprisonment), and it substantially reduces the associated health and social costs. For instance, in the United Kingdom, Gossop et al. (2001) estimated a return of GBP 3 for every additional GBP 1 spent on treatment, as a result of cost savings associated with lower direct and indirect crime-related costs.

Health and social policy programmes have a medical-response component (dealing with a medical need) and a policy-response component (measures aiming to change individual behaviour) (Pacula et al., 2009). It is estimated that around 25 % of Europe's adult population have ever used an illicit drug and that, in 2011, at least 1.2 million people received some kind of treatment for illicit drug use in the EU and its candidate countries (EMCDDA, 2013).

Health expenditure, which includes drug abuse and dependence treatment, is largely public in nature in the majority of OECD countries (OECD, 2015, p. 170). Providing good-quality services for drug users is therefore a significant challenge, particularly in a difficult economic climate, and governments face increasing pressure to monitor their costs.

Research is under way to estimate expenditure — and, in particular, public expenditure — on illicit drug treatment in several countries. However, most researchers are

hampered by a lack of detailed data, as often data — when available — do not make a distinction between expenditure on drugs and on alcohol or mental disorders (Ramstedt, 2006; Rigter, 2006). There have also been attempts to make cross-country comparisons (Lievens et al., 2014), which are even more difficult, as data are often not available.

Portugal is an example of a country in which substance abuse and dependence are mostly treated with public funds. The National Strategy for the Fight Against Drugs, approved by the Portuguese government in 1999, is based on a health-oriented rationale and encompasses various policy measures, including, from 2000 onwards, the decriminalisation of illicit drug possession and consumption. In particular, it also includes an extension of the healthcare services network, a syringe exchange programme, an increase in scientific research funding and specialist training, and a significant financial budget increase for drug-related problems. It led to the setting up of the Portuguese Institute for Drugs and Drug Addictions (IPDT), a public organisation with several responsibilities. In particular, from 2005 onwards and after the merger of the IPDT with the SPTT <sup>(2)</sup>, the Portuguese Institute for Drugs and Drug Addictions (*Instituto da Droga e da Toxicodependência*, known as IDT) became responsible for the drug-related healthcare treatment network and for the elaboration and implementation of the National Action Plan Against Drugs and Drug Addiction <sup>(3)</sup>.

In order to pursue these objectives, an innovative organisational model was introduced. Small treatment teams, belonging to integrated response centres, provide services associated with prevention, harm reduction, social reintegration and treatment. Some of these services are outsourced (e.g. harm reduction, a significant percentage

<sup>(1)</sup> EMCDDA (2014, p. 70) shows that a larger share of drug-related public expenditure is allocated to drug supply reduction activities (as opposed to demand reduction) in most of the 16 European countries that have detailed public expenditure breakdowns.

<sup>(2)</sup> *Serviço de Prevenção e Tratamento da Toxicodependência*.

<sup>(3)</sup> The 2005-12 national plan detailed policy objectives for specific periods in the following areas: prevention; harm reduction and risk minimisation; treatment; social reintegration; combating illicit drug trafficking and money laundering; research, statistical and epidemiological information; evaluation; international collaboration; legal regulation; and decriminalisation consumption.

of prevention- and treatment-related services), but those costs are allocated to the relevant treatment team (e.g. in the case of treatment services, the treatment team that has referred the patient) <sup>(4)</sup>. In addition, some of these services are provided at the individual level (e.g. treatment), but others may be provided to larger groups (e.g. prevention activities targeting specific groups or communities) and, naturally, the associated costs may be very different. A major advantage of this organisational model, from a research perspective, is that it keeps track of costs at the treatment team level, thus generating a rich and useful database of treatment team costs and outputs or activities carried out.

Relying on this IDT cost and output data for its treatment network, during the years 2011 and 2012, we estimated a cost function that allowed us to calculate the costs associated with the various types of dependence-related activities carried out by the various treatment teams across different geographical areas within Portugal. We uncovered some interesting results: for example, the average cost of each prevention event (all substances) is EUR 2 330, while for treatment it is EUR 134 <sup>(5)</sup>. Drug-related activities (including prevention, harm reduction, social rehabilitation and treatment) have an average event cost of EUR 128.

These estimates are quite relevant from a policy point of view. First, they provide an indication of how costly are the various types of activity carried out by treatment teams. Second, they allow for more informed decisions if some of these activities were to be further outsourced (e.g. to non-profit or private healthcare organisations) <sup>(6)</sup>. In addition, although we did not pursue this line of research in this chapter, it also allows for an analysis of the (possible) existence of economies of scale or scope. This is relevant in terms of understanding the adequacy of the treatment network currently in place. More broadly, in the drug dependence field, this methodology can be used by other countries that have geographically decentralised treatment teams to carry out an estimation of their treatment costs. Naturally, depending on what costs are borne by each treatment team, the estimates may vary from country to country, reflecting not only cost differences, but also differences in each country's organisational structure for the treatment of addictive behaviour.

The chapter is organised in the following way: a brief overview of cost functions, a description of the data used, a results section and a conclusion. Annex A provides a detailed, technical description of the methodology used.

## Cost functions: a brief overview

In a nutshell, an organisation or a firm produces (possibly multiple) outputs (e.g. products or services) making use of inputs (for instance, staff or raw materials). For a given choice of output levels, the organisation or firm is typically expected to be economically efficient, that is, to choose the combination of inputs that minimises its costs. The concept of a cost function embodies this notion. In particular, the cost function allows the identification of the minimum possible costs that an organisation or firm must incur to produce a given level of outputs.

Estimating a cost function allows a researcher to understand how changes in output levels may change production costs. Therefore, it is a particularly important concept when organisations or firms wish to depart from their current production levels, as it allows the cost implications of such decisions to be calculated. Such functions can be estimated using data on total costs, output levels and input prices.

## Data

Treatment teams report their costs on an annual basis to IDT, broken down into several cost categories, namely staff costs (including all relevant subsidies or additional remuneration on top of salary costs) and acquisition of services and supplies (clinical material — including methadone, for substitution therapies — food, communications, transport, insurance, security, etc.), which also includes patient referral costs within the drug treatment network, that is, the cost associated with treating a particular treatment team's client at another drug-related healthcare provider (e.g. inpatient treatment in therapeutic communities) <sup>(7)</sup>. Therefore, once a client is admitted by a treatment team, the costs of all services provided to that client (either within the treatment team or, through referral,

<sup>(4)</sup> In the case of social reintegration, treatment teams are responsible for only some activities (and their underlying costs), e.g. social service appointments (social situation diagnosis and referral), targeted interventions, e.g. to increase social or other competences. Other concrete measures for social reintegration (e.g. housing, employment, etc.) are carried out by social security services and their costs are not included in the analysis.

<sup>(5)</sup> As we will later clarify, the unit used for each type of activity is an 'event' or 'episode' registered in the treatment network database.

<sup>(6)</sup> Law no 7/97 allows this outsourcing to occur and, indeed, as mentioned above, this possibility is already used for some activities (e.g. prevention, harm reduction or treatment).

<sup>(7)</sup> Although a broader referral concept is legally possible — through which users under treatment could be referred to other health providers, e.g. for diagnostic examinations or other procedures — it was never adopted by IDT insofar as it would be questionable whether such referrals would be related to the addiction problem.

by other drug-related healthcare providers) are typically allocated to that treatment team.

Output levels were extracted from SIM <sup>(8)</sup>, activity management software used by all treatment teams when registering their activity. This software registers a wide variety of information for each 'event', which is associated with a particular client, namely specific information on each service area (medical, psychological, nursing services) or the main substance to which it refers (alcohol, illicit drugs, tobacco, etc.) <sup>(9)</sup>. In addition, event-specific information is also registered, namely whether or not it was scheduled, whether or not the event actually took place (e.g. not all scheduled events actually take place) and the type of activity to which it refers (e.g. prevention, harm reduction).

We collected this data for the years 2011 and 2012, as during that period the data collection procedure was broadly consistent. From 2013 onwards, IDT became SICAD <sup>(10)</sup> and its responsibilities changed significantly, which ultimately translated into significant differences in the functioning of treatment teams and particularly in the way costs were registered. An important implication of this change is that extending the analysis we carried out to subsequent years (2013 onwards) would not be straightforward and would (almost certainly) involve a lengthier and more intensive data collection exercise. Summary statistics for each of the main variables are presented in Table 11.1.

TABLE 11.1  
Descriptive statistics of the main variables used in the regressions

Variable	No of observations	Mean	SD	Minimum	Maximum
Total costs (EUR)	83	1 170 063	912 894	22 594	4 649 957
Number of staff	73	15	9	1	42
Number of events	83	10 879	6 338	1 273	31 983
Number of treated individuals	83	799	455	166	2 611
Average event duration (minutes)	83	29	5	15	46
<i>Model 1 outputs (number of events)</i>					
Prevention	83	117	154	0	811
Harm reduction	83	124	672	0	6 070
Social rehabilitation	83	1 452	1 324	45	7 137
Treatment	83	9 157	5 765	143	30 786
Other	83	29	92	0	611
<i>Model 2 outputs (number of events)</i>					
Alcohol	83	1 098	908	69	4 382
Drugs	83	9 548	5 967	1 200	29 334
Tobacco	83	34	75	0	467
Other	83	200	226	0	1 273
<i>Model 3 outputs (number of events)</i>					
Alcohol	83	1 098	908	69	4 382
Opiates	83	6 478	3 699	848	17 910
Stimulants	83	454	410	0	1 786
Hallucinogens	83	5	10	0	45
Cannabis	83	302	231	12	1 081
Other drugs	83	2 309	2 706	65	15 372
Tobacco	83	34	75	0	467
Other	83	200	226	0	1 273

Note: 'Number of observations' refers to the number of treatment teams for which data were available; 'Mean' refers to the mean of a given variable across treatment teams; number of staff was not available for all 83 treatment teams.

<sup>(8)</sup> Sistema de Informação Multidisciplinar.

<sup>(9)</sup> An 'event' is the broad term we use to define an activity registered in SIM. For example, a treatment event is usually associated with face-to-face contact for a specific purpose (e.g. an appointment, a blood test, a psychological evaluation). Therefore, on a specific day when interacting with the treatment network, a client may trigger more than one event in SIM.

<sup>(10)</sup> Serviço de Intervenção nos Comportamentos Aditivos e nas Dependências.



- In model 1 (five outputs), as the number of prevention events increases by 1 %, cost increases by 0.12 %; for treatment events, the corresponding cost increase is 1.22 % — therefore, costs are rather sensitive to the number of treatment events.
- In model 2, only two outputs (out of four) have statistically significant cost elasticities. For example, the cost elasticity of illicit drug events is 0.73, while that of other substances is 0.09; therefore, costs are (in relative terms) more sensitive to illicit drug-related events;
- Results from model 3 are more difficult to interpret <sup>(13)</sup>.

Two cost measures can be calculated using the estimates presented in Table 11.2 (see Annex A). First, we can calculate average incremental costs. These are equivalent, in this type of multi-output setting, to an average cost, that is, on average, how much it costs to produce each unit of a given output. This cost measure is a simple indicator of how much, on average, each unit of output costs to produce. Second, the marginal cost of an output tells us how much total costs change when (assuming all else is constant) an additional unit of a given output is produced. This is likely to be different from the average incremental cost, because in order to produce an additional unit it may be that the total cost increases by less than the average incremental cost. For example, it may be that this additional unit requires not a significant increase in fixed or quasi-fixed costs (e.g. number of staff) and only an increase in variable costs. Therefore, evaluating all variables at their sample means, we find that:

- In model 1, the average incremental cost of prevention events is EUR 2 330, while the marginal cost is EUR 1 206; for treatment events, the respective figures are EUR 134 and EUR 164.
- In model 2, the average incremental cost of drug-related events is EUR 128 while the marginal cost is EUR 93.
- In model 3, the average incremental cost of stimulant-related events is EUR 2 687 while the marginal cost is EUR 1 913.

<sup>(13)</sup> Because of the significantly larger number of variables included in the regression, the results of model 3 are more difficult to interpret. For example, only two cost elasticities are statistically significant — opiates and stimulants — but only the latter is positive. In addition, the cost elasticities of alcohol or tobacco-related events are quite different from those obtained in model 2.

TABLE 11.3

**Marginal cost estimates of models 1 and 2**

	Output	Marginal cost (EUR)
Model 1	Prevention	1 206
	Harm reduction	-1 274
	Social rehabilitation	98
	Treatment	164
	Other	-2 076
Model 2	Alcohol	40
	Drugs	93
	Tobacco	941
	Other	563

Although these are most plausible results, we present all the marginal cost estimates for models 1 and 2 in Table 11.3 <sup>(14)</sup>.

## Conclusion

This paper has addressed a little-explored topic: the costs associated with the treatment of substance abuse. Owing to its particular characteristics — namely the fact that a geographically spread treatment network reports costs and output levels to a single public organisation, IDT — we carried out a cost function estimation that allowed us to empirically estimate the costs (average incremental costs and marginal costs) associated with the treatment of substance abuse. We found that prevention and especially treatment appear to be the most cost-sensitive activities (measured by their cost elasticities) carried out by the treatment network. Looking in particular at the treatment cost elasticity, we found that a 1 % increase in the number of treatment events would result in an overall cost increase of 1.22 %. In addition, illicit drugs have a relevant and significant cost elasticity — more so than other substances.

These estimates have (at least) three immediate practical uses. First, within each treatment team, they may be used as a tool for budgeting — that is, predicting yearly costs on the basis of expected output levels. Second, they may be used as benchmarks to identify inefficiency — for instance, if the observed average cost of an activity is much larger than that predicted, it may be because of inefficiencies in service provision, which may then be corrected immediately. Third, these estimates may also be used as benchmarks if some of these activities were to be further

<sup>(14)</sup> Average incremental costs rely on evaluating the cost function far away from the approximation point (when one output level is evaluated at zero) and are thus more sensitive to estimation problems than marginal costs.

outsourced — they provide an indicator of how costly it is to provide a given service within the treatment network and, therefore, may be used as a cap or reference value when contemplating the possibility of outsourcing such services to not-for-profit or private sector providers.

Further research in this area is warranted. This methodology allows the analysis of economies of scale or scope within the treatment network, although we did not pursue it in this chapter. These data also allow a more detailed efficiency analysis, in which comparisons could be made of the outputs produced by each treatment team with the available inputs (e.g. staff). In addition, data for more years or additional variables that can explain the costs of the treatment teams would certainly improve the results of the estimation and thus provide a more accurate calculation of average incremental costs as well as marginal costs. These are likely to be the next steps in our research.

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## Annex A

### Methodology: cost function estimation

A firm's long-run cost function indicates the minimum cost at which a firm produces a given quantity of its various outputs ( $y_i$ ) for given input prices ( $w_j$ ). Under the assumption of  $n$  outputs and  $m$  inputs, a firm's long-run cost function is typically given by:

$$C = C(y_1, \dots, y_n, w_1, \dots, w_m) \quad (\text{A11.1})$$

We assume that treatment teams operate in the long run, that is, we explicitly assume that they can change the quantity they use of all the production inputs in response to changes in input price or output level. This strikes us as a plausible assumption because (1) treatment teams are typically small and appear to make limited use of inputs that might be considered fixed in the short run (and hence whose quantity would not change in response to input price or output level changes), and because (2) treatment teams can refer clients to other drug treatment providers with relative ease, thus effectively allowing possible short-run input constraints to be easily bypassed.

We use the generalised translog cost function to represent the long-run cost function. This cost function is particularly suited when a significant number of observations contain zero values for some output categories. The main difference with respect to the translog cost function is that all output levels are subjected to a Box-Cox transformation instead of the log-transformation commonly used under the translog cost function, that is, all output levels  $y_i$  are transformed into  $Y_i = \frac{y_i^\lambda - 1}{\lambda}$  <sup>(15)</sup>. In addition, prior to the Box-Cox transformation of the output data ( $y_i$ ), we mean-scale all our variables <sup>(16)</sup>.

The generalised translog cost function is a second-order Taylor approximation to the true (but unknown) functional form and it is given by:

$$\begin{aligned} \ln C = & \alpha_0 + \sum_{i=1}^n \beta_i Y_i + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \beta_{ij} Y_i Y_j + \sum_{i=1}^m \gamma_i \ln(w_i) + \\ & + \frac{1}{2} \sum_{i=1}^m \sum_{j=1}^m \gamma_{ij} \ln(w_i) \ln(w_j) + \sum_{r=1}^n \sum_{i=1}^m \delta_{ri} Y_r \ln(w_i) \end{aligned} \quad (\text{A11.2})$$

In line with the literature, we assume a symmetry constraint  $\beta_{ij} = \beta_{ji}$ , and  $\gamma_{ij} = \gamma_{ji}$ , as well as linear homogeneity in input prices (i.e. doubling the price of all inputs leads to a doubling of costs):

$$\begin{aligned} \sum_{i=1}^m \gamma_i &= 1 \\ \sum_{i=1}^m \delta_{ri} &= 0, r = 1, \dots, n \\ \sum_{j=1}^m \gamma_{ij} &= 0, i = 1, \dots, m \end{aligned} \quad (\text{A11.3})$$

<sup>(15)</sup> We assume that  $\lambda=0.1$ .

<sup>(16)</sup> For each output ( $y_i$  ( $i=1, \dots, n$ )) and for each input price ( $w_j$  ( $j=1, \dots, m$ )), we divide each observation by the respective mean. Therefore, the mean of the mean-scaled variables is equal to 1.

The cost proportion equations are obtained through the logarithmic differentiation of the cost function (Shephard's lemma):

$$S_i = \frac{\partial \ln C}{\partial w_i} = \gamma_i + \sum_{j=1}^m \gamma_{ij} \ln(w_j) + \sum_{r=1}^n \delta_{ri} Y_r, i = 1, \dots, m \quad (\text{A11.4})$$

where  $S_i = \frac{w_i x_i}{\sum_{i=1}^m w_i x_i}$  is the cost share of input  $i$  ( $x_i$  represents the quantity used of input  $i$ ).

A key question in our estimation is the definition of outputs for the treatment teams. Indeed, treatment teams have various functions and their work covers a wide variety of areas. As such, it is not easy to define what their outputs are. Therefore, we have estimated three different models, each of which considers a different type of output for treatment teams:

- Model 1: outputs are considered to be activity based, namely we assume that treatment teams provide services in the areas of prevention, harm reduction, social rehabilitation, treatment or other areas.
- Model 2: outputs are considered to be substance based (in broad categories), that is, treatment teams are assumed to provide services associated with alcohol, illicit drugs, tobacco or addiction to other substances.
- Model 3: as in model 2, outputs are considered to be substance based, but illicit drugs are further broken down into opiates, stimulants, hallucinogens, cannabis or other drugs (alcohol, tobacco and other substances are considered, as in model 2, as broad categories).

It strikes us as plausible to assume that all treatment teams rely essentially on two inputs: staff and other costs (which include services and acquisition of supplies). Staff costs were calculated in the following way: for each treatment team, we know how many people in each staff category there are (doctor, nurse, administrative staff). Using the annual salary of each staff category for that year, we can compute an average salary per treatment team<sup>(17)</sup> <sup>(18)</sup>. This is clearly an imperfect measure of staff unit costs. Ideally we would use total staff costs divided by the number of staff, but it appears as if several treatment teams have significant discrepancies in their overall staff costs when compared with the number of staff they report, possibly because of cost allocation errors. In the face of this problem, our proposed method appears more reliable.

Our second input — other costs — is essentially a composite of various input categories. As such, we assume that the price of this input is the result of the division of its total costs by the number of effective events registered by each treatment team (see below). Therefore, as in Garcia and Thomas (2001), this unit price is expressed as a cost per unit of output.

Equations A11.2 and A11.4 were estimated with the homogeneity restrictions of equation A11.3 using Zellner's seemingly unrelated regression (SUR) technique. Because the cost proportions add up to unity, only one of them is independent, and we have thus dropped the second cost proportion equation (associated with other inputs) from the regression. It is immaterial which cost proportion equation is dropped, but input prices are not readily available for the second input. In addition, given the relatively low degrees of freedom in some models (namely model 3), we have estimated all models under the assumption of homotheticity, that is, we assume that the cost-minimising mix of inputs is not affected by

<sup>(17)</sup> Treatment teams' staff are public servants and, as such, their salaries are defined according to a payscale.

<sup>(18)</sup> For a small number of teams, we did not have the staff mix. In this case, we have assumed these units to have the average staff mix in the sample.

the volume or mix of outputs, which implies that changes in input prices affect costs only by a scale factor (Smet, 2002). This implies that, in equation A11.2, input prices are not interacting with output levels.

In order to eliminate potential outliers, we excluded from the analysis observations whose event unit cost (total costs divided by the total number of effective events) was in the top or bottom 5 % (eight observations in total).

Define  $\eta_i$  as the cost elasticity of output:

$$\eta_i = \frac{\partial C}{\partial y_i^*} \frac{y_i^*}{C} \quad (\text{A11.5})$$

That is,  $\eta_i$  represents the percentage change in costs when output  $i$  varies by 1 %. Following Vita (1990), the cost elasticity of output when we use the generalised translog cost function is given by:

$$\eta_i = \left( \beta_i + \sum_{j=1}^n \beta_{ij} Y_j + \sum_{j=1}^m \delta_{ij} \ln(w_j) \right) y_i^\lambda \quad (\text{A11.6})$$

where  $y_i$  are the untransformed outputs and  $\lambda$  is the Box-Cox transformation parameter. Because all variables are mean-scaled, at the sample mean the cost elasticity of output is simply given by  $\eta_i = \beta_i$ .

The average incremental cost of output  $i$  is equivalent, in a multi-output setting, to an average cost. It provides an indication of how much, on average, each unit of output  $i$  costs to produce and it is calculated in the following way (see Grannemann et al., 1986, for example):

$$AIC_i = \frac{C(y_1, \dots, y_n) - C(y_1, \dots, y_{i-1}, 0, y_{i+1}, \dots, y_n)}{y_i} \quad (\text{A11.7})$$

It is based on the incremental cost of output  $i$ , that is, the difference in overall costs between producing all  $n$  outputs and producing all the outputs except  $i$ .

The marginal cost of output is the variation in total costs when (assuming all else is constant) an additional unit of output is produced and is given by:

$$MC_i = \frac{\partial C}{\partial y_i} = \frac{C}{y_i} \frac{\partial \ln C}{\partial Y_i} = \frac{C}{y_i} \left[ \beta_i + \sum_{j=1}^n \beta_{ij} Y_j \right] \quad (\text{A11.8})$$