Bank Risk-Taking and Impaired Monetary Policy Transmission*

Philipp J. Koenig[†] and Eva Schliephake[‡]

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Abstract

We consider a standard banking model with agency frictions to simultaneously study the weakening and reversal of monetary transmission and banks' risk-taking in a low-interest environment. Both, weaker monetary transmission and higher risk-taking arise because lower policy rates impair banks' net worth. The pass-through to deposit rates, the level of excess reserves and the extent of the agency problem between banks and depositors are crucial determinants of monetary transmission. If the deposit pass-through is sufficiently impaired, a reversal rate exists. For policy rates below the reversal rate further interest rate reductions lead to a disproportionate increase in risk-taking and a contraction in loan supply.

Keywords: Monetary policy, Bank lending, Risk-taking channel, Reversal rate

JEL Classifications: G21, E44, E52

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[†]Research Centre, Deutsche Bundesbank; philipp.koenig@bundesbank.de

[‡]Universidade Católica Portuguesa, Católica Lisbon School of Business & Economics, Portugal, schliephake@ucp.pt (Corresponding author)

1 Introduction

For the past decade, central banks in advanced economies have engaged in unprecedented monetary easing to stimulate bank lending and boost aggregate demand. This prolonged period of loose monetary policy raises concerns about its potential undesirable consequences. First, loose monetary policy may induce banks to increase risk-taking, which could pose a threat to financial stability (Borio & Zhu, 2012). Second, reductions in central bank policy rates could depress bank profits to the point that banks would respond to further monetary stimulus by raising, rather than lowering, loan rates, thus choking off the credit supply to the economy (Brunnermeier & Koby, 2017).

In the present paper, we propose a stylized model that allows to study the risk-taking and the impaired bank lending channels simultaneously. Our model generates predictions that are in line with the empirical findings with regard to the effects of monetary policy in low interest environments. In particular, we show that higher risk-taking incentives and the impairment of the bank lending channel can be viewed as two sides of the same coin: both arise in response to an adverse effect of lower policy rates on banks' profitability.

The extant literature has studied the risk-taking channel and the impairment of the bank lending channel in separate theoretical frameworks. Models of the risk-taking channel (e.g., Dell'Ariccia et al. (2014); Martinez-Miera & Repullo (2017)) can offer explanations for the empirically observed negative relationship between bank risk-taking and policy rates (Maddaloni & Peydró, 2011; Dell'Ariccia et al., 2017). However, these models abstract from the characteristics of the current low interest environment. Therefore, they cannot explain empirical observations that occur specifically at low levels of policy rate, e.g., a positive relationship between bank profits and policy rates (Ampudia & Van den Heuvel, 2019; Wang et al., 2020), or a negative relationship between mortgage rates and policy rates (Basten & Mariathasan, 2020; Miller & Wanengkirtyo, 2020). These empirical relations are implied by models that study the impaired bank lending channel and emphasize the importance of the lower bound on deposit rates and banks' excess liquidity holdings (e.g., Brunnermeier & Koby (2017); Eggertsson et al. (2019); Ulate (2021)). However, these models abstract from banks' risk and therefore can explain neither why banks may increase their risk-taking when policy rates become negative (Basten & Mariathasan, 2020; Heider et al., 2019; Bittner et al., 2021), nor why a weaker pass-through to loan rates can be observed specifically for riskier banks (Arce et al., 2021).

We consider a model of financial intermediation in which a banker uses deposits and own equity to fund the issuance of risky loans and holdings of safe reserves. The banker can exert monitoring effort to reduce the default risk of her loan portfolio. While depositors can observe the banker's choice of loan issuance and reserve holdings, her monitoring effort is unobservable, inducing an agency problem between the banker and her depositors.

We make two assumptions that reflect key characteristics of low interest rate environments. First, there is a lower bound on deposit rates; i.e., there exists a minimal return that the bank must offer on deposits for agents to be willing to hold them rather than switch to cash. This assumption reflects the empirical observation that deposit rates tend to approach a lower bound as changes in deposit rates become progressively smaller when policy rates are lowered into negative territory (Eggertsson et al., 2019). Second, the banker holds excess reserves. That is, we assume that even when policy rates become rather low, the banker never uses up her entire funding base to fund loan issuance and always retains some holdings of liquid reserves with the central bank. This assumption reflects the fact that since the financial crisis of 2008/09, reductions in short-term policy rates have been accompanied by an increase in large-scale asset purchase and lending programs by central banks. These measures have increased banks' reserve holdings in excess of regulatory provisions (such as minimum reserves and liquidity requirements).

In our model, monetary transmission, i.e., the effect of the risk-free rate on loan rates and volumes, works via two channels: A portfolio adjustment channel and, a risk channel. The portfolio adjustment channel reflects the conventional view of monetary transmission whereby lower risk-free rates are expansionary and translate into more bank lending. A lower risk-free rate reduces the return on risk-free reserves and thereby the opportunity cost of loan issuance. The banker optimally balances the lower cost of lending by decreasing the loan rate and increasing the loan supply.

The risk channel arises because the agency problem between the banker and her depositors implies that the banker's monitoring incentives are directly affected by changes in the risk-free rate. First, a lower risk-free rate reduces the profitability of reserves and thus lowers total profits. This *reserve earnings effect* worsens monitoring incentives. Second, when the banker can pass a lower risk-free rate on to depositors, profits increase. This *deposit pass-through effect* improves monitoring incentives. When the first effect dominates the second effect, the banker's monitoring incentives decline when the risk-free rate falls. This negative effect of lower risk-free rates on monitoring incentives can arise in our model because of the lower bound on deposit rates. As a consequence, when the reserve earnings effect dominates the deposit pass-through effect, further reductions in the risk-free rate worsen monitoring incentives, and the banker optimally increases her risk-taking.

To balance the adverse effect of a higher risk level on profits, the banker has an incentive to optimally increase the loan rate and reduce the loan volume. We show that the risk channel counteracts the portfolio adjustment channel and that the effect of lower policy rates on the banker's loan issuance becomes weaker if the risk-free rate falls below a critical value.

Furthermore, we show that the risk channel can also dominate the portfolio adjustment channel. In particular, we show the existence of a critical value below which further reductions in the risk-free rate induce banks to cut back on lending. The critical value constitutes a reversal rate, as in Brunnermeier & Koby (2017). Like their model, a precondition for the reversal rate in our model is that bank profits decline when the risk-free rate falls. In contrast to Brunnermeier & Koby (2017), however, the reversal rate in our model does not arise due to an exogenous constraint on future bank profits but due to the agency friction between the banker and her depositors.

We extend our model to consider the effects of insured deposits and perfect competition on the possibility of transmission reversal. Deposit insurance (not fairly priced in equilibrium) provides an (exogenous) subsidy to the bank that increases its profits at any risk-free rate. As a consequence, deposit insurance mitigates the problem of transmission reversal. At the limit, when all deposits are insured, the reversal rate ceases to exist. Thus, ceteris paribus, transmission reversal is less of a problem for banks that are funded with a larger share of insured deposits. Perfect competition, in contrast, can amplify the problem of transmission reversal. Compared to the case with loan market power, the condition for a reversal is weaker under perfect competition. This is because competition erodes profits such that banks may be forced to reduce loan issuance at a higher level of the risk-free rate.

In our baseline model, we make a strict assumption to ensure that the bank holds a strictly positive reserve balance. We justify this assumption by making reference to the current environment of high excess liquidity in the banking sector which is primarily driven by central banks' asset purchase programs. In an extension, we show that our main results remain largely unchanged if we allow the bank to borrow from the central bank, i.e., hold negative reserve balances, but deposits are subject to random in- and outflows.

Related Literature. Our paper relates to a large body of literature that analyzes the transmission of monetary policy via the banking sector. The traditional view of the bank lending channel holds that a reduction in policy rates reduces banks' funding cost and induces greater loan supply (Bernanke & Blinder, 1988; Bernanke & Gertler, 1995; Kashyap & Stein, 1994). While this channel is also at work in our model, we show that it can be weakened or amplified by a risk channel that arises due to agency frictions between the bank and its depositors.

This relates our paper to the literature on the bank risk-taking channel, which argues that low policy rates may lead banks to increase the riskiness of their loan portfolio (Gambacorta, 2009; Rajan, 2010; Borio & Zhu, 2012). In particular, our model builds on Dell'Ariccia et al. (2014, 2017), who emphasize the role of agency frictions between banks and their creditors as a key determinant of the risk-taking channel. The bank in our paper faces a similar agency problem as banks in Dell'Ariccia et al. (2014) or Cordella et al. (2018). Furthermore, Martinez-Miera & Repullo (2020) show that the loan market structure is key for the relationship between interest rates and risk-taking decisions of financial intermediaries because it shapes the extent to which lower funding costs are passed through to loan rates. The ability of banks to pass lower risk-free rates through to deposit and loan rates is also a driver in our model. However, the former models assume that the pass-through of monetary policy to deposit rates is frictionless. In contrast to these papers, we focus on monetary transmission in a low interest rate environment and take into account the effects of reserve holdings and an imperfect pass-through to deposit rates.

This connects our paper to the growing literature on monetary policy transmission in a low interest rate environment. Eggertsson et al. (2019) argue that due to the increasing attractiveness of cash, the pass-through to deposit rates becomes impaired when the policy rate approaches the zero lower bound or becomes negative. Brunnermeier & Koby (2017) show the existence of an effective lower bound – the so-called reversal rate – below which further reductions in policy rates lead to an increase in loan rates. Both, Brunnermeier & Koby (2017) and Eggertsson et al. (2019) derive their theoretical results by imposing an exogenous net worth constraint that, once binding, mechanically increases the equilibrium loan rate. Repullo (2020) points out that such an exogenous constraint on the future value of the bank's capital does not reflect standard banking regulation. Repullo (2020) further demonstrates that the reversal rate fails to arise in a model with a standard capital requirement. Our paper complements this literature by tying the net worth constraint to a standard agency problem between the banker and her creditors. In this respect, our paper is also related to Pariès et al. (2020), who study the reversal rate in a macroeconomic general equilibrium model and consider the bank's net worth constraint as an agency problem. However, the net worth constraint in Pariès et al. (2020) arises because the banker can abscond with the deposits rather than because of unobservable monitoring and risk-taking, as in our model. Thus, while they also conclude that larger bank equity mitigates the reversal problem, they abstract entirely from risk-taking.

2 Model setup

We consider a bank over two periods, indexed by $t = 0, 1$. The bank is run by a risk-neutral bank owner/manager ("banker") who is endowed with own funds and receives deposits from a large number of risk-neutral depositors. The banker decides on the issuance of loans and on the monitoring of her loan portfolio. Monitoring entails a private cost for the banker and reduces the riskiness of her loan portfolio. Depositors cannot observe the banker's monitoring choice, and the banker cannot commit to a certain level of monitoring. The main focus of our analysis is on the transmission of monetary policy to loan rates, the loan volume and the banker's monitoring/risk taking. We take the gross risk-free interest rate $r > 0$ as the measure of monetary policy, and we assume that it can be perfectly controlled by the central bank.

Bank liabilities. The banker is endowed with own funds $E \geq 0$. In addition, she can attract deposits in period 0 from a large number of depositors. Deposits are uninsured and depositors need to be compensated for the risk that the banker becomes unable to fully repay the depositors in period 1.[1] Thus, to attract deposits, the banker must offer a deposit rate, r_D, that, in expected terms, matches depositors' outside option, $u(r)$.

Assumption 1. *The depositors' outside option* $u(r) \geq r$ *is bounded below by* $\underline{u}$. *For* $r > u^{-1}(\underline{u})$, $u(r)$ *is strictly increasing* ($u'(r) > 0$) *and weakly convex* ($u''(r) \geq 0$).

The lower bound $\underline{u}$ reflects the idea that depositors would switch to other assets, such as non-interest bearing cash holdings, once the risk-free rate becomes too low. The lower bound is not necessarily equal to zero, as negative rates could still be compensated for in the form of non-pecuniary benefits of deposits, such as the safety and ease of making payments.[2] The further assumption that the outside option weakly increases at an increasing rate reflects the idea that the relative benefit of holding deposits decreases in an environment of higher real interest rates when the profitability of other assets increases.

To simplify the exposition in the main part of this paper, we assume that the banker cannot adjust the 'intensive margin' of her deposits. That is, she either raises an amount D or no deposits at all. Our preferred interpretation of this assumption is that deposits are subject to in- and outflows that cannot be easily scaled by the

[1] Section 4.1 considers the effect when the banker also issues insured deposits.

[2] For example, suppose that depositors, instead of holding deposits, could either hold a risk-free bond at rate r or cash which provides a per-unit convenience yield θ and requires a per-unit storage cost x. For this case, $u(r) = \max\{1 + \theta - x, r\}$.

banker because they are determined by the decisions of depositors. For example, depositors can be firms that hold their working capital with the bank and need to make payments. Alternatively, deposits can be created when the central bank purchases bonds and other fixed-income assets from depositors under quantitative easing programs. When buying such assets from non-bank entities, the central bank transfers the purchase price to their deposit accounts, while the respective banks receive a transfer of central bank liquidity into their reserve accounts. The banker cannot easily scale such flows up or down since they are determined by the depositors' decisions to sell (buy) securities.[3]

Bank assets and monitoring. The banker is a monopolist in the local loan market. Loan demand in period 0 is described by a demand curve $L(r_L)$, with $L'(r_L) < 0$ and $L''(r_L) \leq 0$, where r_L denotes the gross interest rate that the bank charges on loans.[4] Loans are risky and are repaid in period 1 with probability $q \in (0, 1)$. The banker can exert unobservable monitoring effort to influence the repayment probability of her loan portfolio. We assume that monitoring is translated one-to-one into the success probability so that the banker can directly choose q. Monitoring entails a private cost

$$c(q) = \frac{\kappa}{2} q^2, \quad \text{where } \kappa > 0.$$

[3]The current level of excess reserves by the banking sector is primarily driven by the quantitative easing (QE) programs of central banks. For example, since 2015, the euro area banking sector increased reserve holdings by more than EUR 1.3 trillion (Bechtel et al., 2021).

[4]As a technical condition to ensure the uniqueness of the equilibrium, we assume that $L(\cdot)$ is not too concave, i.e., $|L''(r_L)|$ is bounded above.

Alternatively, the banker can invest in a risk-free asset that yields the gross risk-free return r in period 1. For example, this can be current accounts held with the central bank or investments in high-quality government bonds. The amount invested in the risk-free asset is denoted by R and henceforth referred to as reserves.

The bank's funding constraint in period 0 is given by

$$R + L = D + E. \tag{1}$$

The level of reserves is endogenously determined through the loan choice of the banker as the residual $R = D + E - L$. Henceforth, we use $\rho \equiv \frac{R}{D} = \frac{D+E-L}{D}$ to denote the share of deposits held in risk-free reserves and we refer to it as the reserves-deposit-ratio.

To simplify the exposition of the main part of the analysis, we focus on the case in which the banker always holds a positive reserve balance.

Assumption 2. *The elasticity of the loan demand function, $\eta(r_L) \equiv -\frac{L'(r_L)r_L}{L(r_L)}$, satisfies*

$$\eta(L^{-1}(D + E)) < 1.$$

Assumption 2 implies that the banker never exhausts her entire funding base to issue loans but always holds a strictly positive amount of excess reserves. The case of large (excess) reserves is currently the empirically relevant case and likely remains so for the foreseeable future. During the 2008/09 financial crisis, major central banks, such as the Eurosystem, the Federal Reserve or the Bank of England, deviated significantly from their previous regimes of neutral liquidity conditions and have not since returned to their pre-crisis modes of liquidity management. In particular, the increase in large-scale asset purchases since the start of the COVID-19 pandemic has further ratcheted up banks' holdings of reserves in excess of regulatory requirements. We relax Assumption 2 in Section 4.3, where we introduce random liquidity shocks to deposits but we allow the bank to choose the deposit volume endogenously and allow the bank to borrow from the central bank, i.e., $R < 0$.

Sequence of events and equilibrium. Figure 1 shows the sequence of events in the model. An equilibrium of the model is given by a loan rate r_L^* and a deposit rate r_D^*, which jointly determine the bank's optimal loan supply, L^*, optimal reserves, R^*, and the monitoring choice, q^*. The loan rate r_L^* and the monitoring choice q^* maximize the banker's expected profits given the funding constraint (1), while the deposit rate r_D^* ensures depositor participation, given depositors' rational expectations about the bank's monitoring choice.

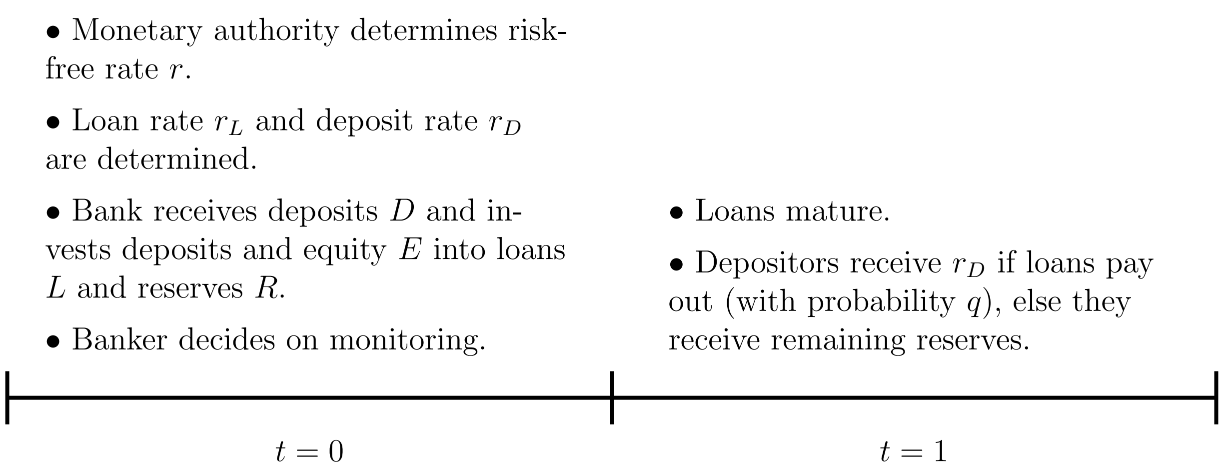

Figure 1: Sequence of Events

3 Monetary transmission with the portfolio adjustment and risk channels

Optimal monitoring choice. We solve the model backwards by first considering the banker's optimal choice of monitoring and then determining her optimal loan issuance and reserve holdings. The banker's expected profits, given r_L and R, can be written as

$$\Pi = q\left(r_L L(r_L) + rR - r_D D\right) - \frac{\kappa q^2}{2} - rE. \tag{2}$$

The first-order condition for the optimal monitoring choice becomes[5]

$$r_L L(r_L) + rR - r_D D - \kappa \hat{q} = 0. \tag{3}$$

Given that depositors rationally anticipate the bank's optimal monitoring choice $\hat{q}$, the interest rate on deposits that ensures depositor participation must satisfy

$$\hat{q} r_D + (1-\hat{q})\frac{rR}{D} \geq u(r). \tag{4}$$

Depositors expect to be paid r_D with probability $\hat{q}$. With converse probability, the bank defaults when loans do not pay out at maturity and depositors obtain a senior claim over a pro-rata share of the remaining assets (reserves plus accrued interest). The expected repayment to depositors must be at least as large as their outside option, $u(r)$. Since the bank's expected profits are strictly decreasing in r_D,

[5] All derivations can be found in the Appendix.

condition (4) binds at the optimum, so we can substitute

$$r_D = \frac{u(r) - (1-\hat{q})\frac{rR}{D}}{\hat{q}} \tag{5}$$

into condition (3) and solve for the optimal monitoring choice $\hat{q}$.[6]

Lemma 1. *The banker's optimal monitoring choice is given by a function $\hat{q}(r_L, r)$ with*

$$\frac{\partial \hat{q}}{\partial r_L}\begin{cases} \geq 0 & \textit{if } \frac{\hat{q}r_L - r}{\hat{q}r_L} \leq \frac{1}{\eta(r_L)} \\ < 0 & \textit{else} \end{cases} \quad \textit{and} \quad \frac{\partial \hat{q}}{\partial r}\begin{cases} \geq 0 & \textit{if } u'(r) \leq \rho \\ < 0 & \textit{else} \end{cases}$$

where $\eta(r_L) \equiv -L'(r_L)r_L/L(r_L)$ denotes the loan demand elasticity and $\rho \equiv R/D$.

The effects of r_L and r on the optimal monitoring level reflect the effects of these rates on the banker's expected profits. Whenever a marginal increase in these rates raises profits, the banker increases her monitoring and *vice versa*.

More specifically, a higher loan rate increases monitoring whenever the Lerner index, $(\hat{q}r_L - r)/\hat{q}r$, is lower than the inverse loan demand elasticity, $1/\eta(r_L)$, which is the standard condition for the profits of a monopolistic bank to increase in r_L (Freixas & Rochet, 1997).

Whether a lower risk-free rate increases profits and therefore leads to higher monitoring depends on the relative magnitude of two effects. On the one hand, a marginal reduction in the risk-free rate lowers the value of the depositors' outside

[6]Since the equation that pins down $\hat{q}$ is quadratic, there are two solutions. Following Allen et al. (2011), we choose the larger of the two roots. Moreover, as Dell'Ariccia et al. (2014), we focus on the interior solution where $\hat{q} < 1$ and abstract from the corner solution where $\hat{q} = 1$. There is a sufficiently large range of values for κ such that the interior solution exists.

option and thereby reduces the banker's expected deposit funding costs. This *deposit pass-through effect* increases profits by an amount $u'(r)D$ and incentivizes the banker to increase monitoring. On the other hand, a marginal reduction in the risk-free rate reduces the banker's return on safe reserves. This *reserve earnings effect* reduces profits marginally by R and induces the banker to reduce monitoring. Thus, a marginally lower risk-free rate decreases monitoring if the deposit pass-through effect is smaller than the reserve earnings effect, i.e., if

$$u'(r)D < R \;\Leftrightarrow\; u'(r) < \rho. \tag{6}$$

Optimal loan issuance and reserve holdings. Substituting the funding constraint (1), the deposit rate (5) and the banker's optimal monitoring choice $\hat{q}(r_L, r)$ into Equation (2) allows us to rewrite expected profits as

$$\Pi = \underbrace{\hat{q}(r_L, r) r_L L(r_L)}_{\text{Expected earnings on loans.}} + \underbrace{r(D + E - L(r_L))}_{\text{Earnings on reserves}} - \underbrace{(u(r)D + rE)}_{\text{Cost of funds}} - \underbrace{\frac{\kappa \hat{q}(r_L, r)^2}{2}}_{\text{Monitoring cost}}. \tag{7}$$

The banker's remaining choice variable is the loan rate r_L. The optimal loan rate, r_L^*, is determined by the standard condition for loan issuance of a monopolistic bank: the Lerner index equals the inverse loan demand elasticity

$$\frac{\hat{q}(r_L^*, r) r_L^* - r}{\hat{q}(r_L^*, r) r_L^*} = \frac{1}{\eta(r_L^*)}. \tag{8}$$

At the optimum point, the loan demand elasticity exceeds unity, $\eta(r_L^*) > 1$. Condition (8) follows from the fact that the effect of r_L on $\hat{q}$ can also be expressed in terms of the Lerner index and the inverse demand elasticity (cf. Lemma 1).

Monetary policy transmission. Monetary policy affects the banker's optimal loan rate (and therefore the loan volume) through a *portfolio adjustment channel* and a *risk channel*

$$\frac{dr_L^*}{dr} = \underbrace{\overbrace{\frac{\partial r_L^*}{\partial r}}^{(+)}}_{\substack{\text{portfolio adjustment}\\ \text{channel}}} + \underbrace{\overbrace{\frac{\partial r_L^*}{\partial q}}^{(-)} \times \overbrace{\frac{\partial \hat{q}(r_L^*, r)}{\partial r}}^{(+)/(-)}}_{\text{risk channel}}. \tag{9}$$

The conventional view of monetary policy transmission holds that monetary policy actions that lower the risk-free rate are expansionary, as they lead to greater bank loan issuance. The portfolio adjustment channel reflects this conventional monetary policy transmission. Effectively, the banker solves an optimal portfolio problem by allocating her funding resources between two investment opportunities (loans and reserves).[7] Given $\hat{q}$, a lower risk-free rate reduces the opportunity cost of investing in loans rather than holding reserves. As a consequence, the banker optimally reduces the loan rate and increases her loan issuance.

In contrast to the portfolio adjustment channel, the direction of the risk channel is ambiguous and it can either amplify or dampen the portfolio adjustment effect. Ceteris paribus, a lower success probability increases the loan rate, thereby reducing the amount of loan issuance, i.e., $\partial r_L^*/\partial q < 0$. Thus, whenever the reserve earnings

[7] Since the volume of deposits is fixed, the optimal loan rate is independent of the costs of deposits as in the textbook version of a monopolistic bank with separable loan and deposit choices (Freixas & Rochet, 1997). In Section 4.3, we show a variant of the model where the bank can choose its deposit volume.

effect dominates the deposit pass-through effect, a reduction in the risk-free rate lowers monitoring, $\partial\hat{q}/\partial r > 0$, and the risk channel counteracts the portfolio adjustment channel, thereby weakening monetary transmission.

Proposition 1. *For a sufficiently small risk-free rate, the reserve earnings effect dominates the deposit pass-through effect and the risk channel weakens the transmission of monetary policy via the portfolio adjustment channel, i.e., there exists* $\overline{r}$ *such that*

$$r < \overline{r} \Rightarrow \frac{\partial\hat{q}}{\partial r} > 0. \tag{10}$$

To understand the intuition behind Proposition 1, note that the lower bound on $u(r)$ implies that for $r < u^{-1}(\underline{u})$, the deposit pass-through becomes fully impaired, i.e., $u'(r) = 0$, and the banker is unable to pass a lower risk-free rate through to her depositors, thereby becoming unable to further reduce her expected funding cost. Moreover, by Assumption 2, the banker always holds a strictly positive level of reserves. Thus, the reserve earnings effect must dominate the deposit pass-through effect already at a level of the risk-free rate $\overline{r} > u^{-1}(\underline{u})$. Below the critical value $\overline{r}$, a lower risk-free rate reduces the banker's monitoring incentives, and the risk channel weakens the portfolio adjustment channel.[8]

Reversal of monetary transmission. The risk channel may not only weaken the portfolio adjustment channel but also dominate it, implying that a lower risk-

[8] Observe that condition (10) is only a sufficient condition. It does not rule out the possibility that the reserve earnings effect becomes dominant over the deposit pass-through effect at a higher level of the risk-free rate (above $\overline{r}$). Whether such a case can arise depends crucially on further properties of $u(r)$ and $L(r_L)$, such as the curvature or magnitude of its rate of change.

free rate will lead to an *increase* in the loan rate and a *reduction* in the bank's loan supply.

Proposition 2. *The risk channel dominates the portfolio adjustment channel, i.e.,* $\frac{dr_L^*}{dr} < 0$, *if and only if*

$$\frac{\partial \hat{q}(r_L^*, r)}{\partial r} \frac{r}{\hat{q}(r_L^*, r)} > 1. \tag{11}$$

To understand the intuition behind Proposition 2, recall that, on the one hand, a lower success probability makes loan issuance relatively less profitable compared to holding reserves, implying that the bank cuts back its loan issuance when q falls. On the other hand, a lower risk-free rate reduces the return on reserves and makes holding reserves less profitable. If the reduction in the risk-free rate lowers the success probability and therefore the profitability of loans by more than the profitability of reserves, the bank prefers to hold more reserves, despite a lower risk-free rate. However, for the profitability of loans to fall by more than the profitability of reserves, the banker's monitoring must react strongly enough, i.e., a reduction in r must lead to a disproportional reduction in $\hat{q}$.

Proposition 3. *If the monitoring costs are sufficiently large, then the risk channel dominates the portfolio adjustment channel if the risk-free rate becomes sufficiently low: i.e., for* $\kappa > \underline{\kappa}$, *there exists a critical value* $\hat{r} < \bar{r}$ *such that*

$$r < \hat{r} \iff \frac{dr_L^*}{dr} < 0.$$

The critical value $\hat{r}$ *is strictly increasing in the bank's monitoring cost,* κ, *and strictly decreasing in the banker's own equity,* E.

Proposition 3 translates the condition in Proposition 2 into a critical value for the risk-free rate. In particular, whenever monitoring costs are sufficiently high and the risk-free rate falls below the critical rate, then the elasticity of $\hat{q}$ becomes sufficiently large so that the risk channel becomes the dominant transmission channel of monetary policy. As in Brunnermeier & Koby (2017), below $\hat{r}$, reductions in the risk-free rate are contractionary, rather than expansionary, and $\hat{r}$ constitutes a *reversal rate*.

However, in contrast to Brunnermeier & Koby (2017), the reversal rate in our model is a consequence of banks' risk-taking behavior. At a sufficiently low level of the risk-free rate, further reductions in the policy rate cannot be fully passed through to depositors. Thus, given large reserve holdings, lower policy rates depress banks' expected profits, thereby increasing their risk-taking incentives and leading them to raise loan rates.

Implications of the model. We use Propositions 1 and 3 to derive implications of the effects of monetary policy in a high excess liquidity and low interest rates environment.

Hypothesis 1. *For given equity E, higher (excess) reserves are associated with*

1. *higher bank risk-taking;*
2. *higher loan rates and a lower loan volume;*
3. *weaker monetary policy transmission.*

Hypothesis 1 follows because larger reserves strengthen the reserve earnings effect compared to the deposit pass-through effect. As a consequence, the threshold $\bar{r}$ increases, and the range of policy rates at which the risk channel weakens the transmission via the portfolio channel becomes larger. Hypothesis 1 is in line with recent empirical findings by Miller & Wanengkirtyo (2020), who study monetary transmission in an environment characterized by excess liquidity. In particular, they show that, following a reduction in the policy rate, banks with larger excess reserves extend lending to riskier borrowers. Moreover, they also find that the transmission of policy rates to loan rates becomes substantially weaker in an environment of excess liquidity, i.e., comparing transmission before and after the financial crisis, they show that the presence of excess liquidity reduces the transmission into loan rates by about 28 bps.[9]

Hypothesis 2. *The reversal rate is larger if, ceteris paribus:*

1. *the bank is endowed with a smaller level of equity;*
2. *the bank is riskier and its loan portfolio is more costly to monitor.*

Hypothesis 2 follows from the effects of equity, E, and the monitoring cost parameter, κ, on the reversal rate $\hat{r}$ (*cf.* Proposition 3). Brunnermeier & Koby (2017) also show that the reversal rate increases with bank equity, but the underlying mechanism in our model is different. In our model, a smaller equity endowment (as well as a larger cost parameter κ) strengthens the bank's agency problem and increases

[9] Similarly, Jimenez et al. (2012) show that banks with more liquidity on their balance sheet expand loan issuance less following a rate cut. However, their data do not allow us to disentangle excess and required reserves.

its risk-taking incentives. Since higher risk-taking raises the loan rate for any value of r, the reversal rate at which the risk channel dominates the portfolio channel also increases.

Hypothesis 2 is in line with recent evidence by Arce et al. (2021), who show that a negative correlation between policy and loan rates can be found for banks that are weakly capitalized and whose lending is riskier. Similarly, Basten & Mariathasan (2020) and Miller & Wanengkirtyo (2020) find that lower policy rates are negatively correlated with mortgage rates, but not with interest rates on other types of loans. To the extent that mortgage handling is relatively more costly than the origination and handling of short-term uncollateralized loans, Hypothesis 2 is consistent with these findings.

4 Extensions

4.1 Insured deposits

In this section, we consider how deposit insurance alters the transmission of monetary policy via portfolio adjustment and risk channels and the possibility of a transmission reversal. Suppose that a share $\delta \in [0, 1]$ of deposits are insured at a zero flat rate. Insured depositors have the same outside option as uninsured depositors, $u(r)$.

As before, we solve the model backwards, by first deriving the banker's optimal monitoring choice and thereafter the optimal loan rate. The first-order condition for the monitoring choice is as in Equation (3), except that we replace the deposit rate r_D by the average deposit rate, $\bar{r}_D$, which depends on the share of insured deposits.

As uninsured depositors rationally anticipate bank monitoring $\hat{q}$, the average deposit rate is [10]

$$\bar{r}_D = \frac{(\delta\hat{q} + (1-\delta)u(r) - (1-\delta)(1-\hat{q})\frac{rR}{D}}{\hat{q}}.$$

Substituting $\bar{r}_D$ into Equation (3) implicitly defines the banker's optimal monitoring $\hat{q}(r_L, r, \delta)$. Importantly, the condition for $\hat{q}$ to increase in r is the same as before in Lemma 1

$$\frac{\partial \hat{q}(r_L, r, \delta)}{\partial r} > 0 \Leftrightarrow u'(r) < \rho.$$

An increase in δ increases monitoring: $\frac{\partial \hat{q}}{\partial \delta} > 0$. This 'charter value effect' of deposit insurance is described in Cordella et al. (2018):[11] As the deposit rate is given when the banker chooses her monitoring, a higher share of insured deposits amounts to a greater implicit subsidy from the deposit insurance, thereby reducing the repayments to depositors and increasing the banker's profits. As a consequence, higher deposit insurance coverage strengthens monitoring incentives.

The banker's expected profit takes the same form as before in Equation (7) except that now the implicit subsidy from funding with a share δ of insured deposits is added. We can use the average deposit rate and the banker's optimal monitoring choice to write expected profits as

$$\Pi \;=\; \hat{q}(r_L, r)L(r_L) + rR - (u(r)D + rE) - \frac{\kappa \hat{q}(r_L, r)^2}{2} + \; S(\delta, r_L, r, R)$$

[10] For simplicity, we assume that, after default at maturity, the bank's cash flows from reserves are split on a *pro-rata* basis among all depositors, insured and uninsured.

[11] Cordella et al. (2018, Proposition 1) show that the effect arises if the share of deposit liabilities that are priced 'at the margin' is sufficiently small. In our model, this share is zero since depositors never observe the banker's actual risk-taking.

where $S(\delta, r_L, r, R) \equiv (1 - \hat{q}(r_L, r, \delta))\delta(u(r)D - rR)$ is the implicit subsidy from the deposit insurance.

Ceteris paribus, larger reserves reduce the implicit subsidy. This is because deposit insurance can use safe reserves to cover (part of) its liabilities in case the loans fail.

The transmission of monetary policy works as before via the portfolio adjustment and risk channels. Since optimal monitoring increases in the risk-free rate whenever the reserve earnings effect dominates the deposit pass-through effect, the condition for the risk channel to weaken monetary transmission remains formally the same as in the benchmark model with $\delta = 0$.

However, the presence of insured deposits changes the relative importance of the portfolio adjustment and risk channels in the transmission of monetary policy.

Proposition 4. *Given a share δ of insured deposits, the risk channel dominates the portfolio adjustment channel, i.e., $\frac{dr_L^*}{dr} < 0$, if and only if*

$$\frac{\partial \hat{q}(r_L, r, \delta)}{\partial r} \frac{r}{\hat{q}(r_L, r, \delta)} > 1 + \frac{\delta \hat{q}}{1 - \delta}.$$

Comparing Propositions 2 and 4 shows that the condition for the dominance of the risk channel is stronger when the banker is funded with insured deposits. The reason is that the banker obtains a larger implicit subsidy from deposit insurance when she holds fewer reserves. This asset substitution motive provides an additional incentive for the banker to increase her loan issuance when the risk-free rate falls. Thus, deposit insurance strengthens the portfolio channel and alleviates the problem of transmission reversal. Simply put, the deposit insurance subsidy mitigates the adverse effect of lower rates on the bank's profitability by increasing its profits.

Hypothesis 3. *The reversal rate $\hat{r}$ is smaller for banks that are funded with a larger share of insured deposits. In the limit, for $\delta \to 1$, the reversal rate ceases to exist.*

4.2 Monetary transmission and competition

We now consider the effect of perfect competition on the weakening and reversal of monetary transmission. We focus again on the baseline case in which deposits are not insured. Following Dell'Ariccia et al. (2014), we assume that the loan rate is set competitively so that the bank makes zero profits in equilibrium.

When solving, given a loan market equilibrium, the monitoring effort is implicitly defined by the same condition as before, which we obtain by combining Equations (3) and (4)

$$\hat{q} r_L L + r\, R - u(r) D - \kappa\, \hat{q}^2 = 0. \tag{12}$$

We then impose a zero profit condition on the bank's loan choice:

$$\Pi \;= \hat{q}(r_L, r) r_L L + r\, R - u(r) D - r\, E - \frac{\kappa\, \hat{q}(r_L, r)^2}{2} = 0. \tag{13}$$

The zero profit condition implicitly defines the equilibrium loan rate r_L^* as a function of the risk-free rate r. Equations (12) and (13) simultaneously define the perfectly competitive banking sector equilibrium.[12]

[12]The representative banker's risk choice is similar to her risk choice under loan market power (cf. Lemma 1), except that the banker's loan choice L is independent of r_L.

Substituting Equation (12) into (13) we obtain the equilibrium condition that pins down the equilibrium loan rate

$$-rE + \kappa\frac{\hat{q}(r, r_L)^2}{2} = 0. \tag{14}$$

The competitive loan rate is implicitly defined by the equality of monitoring effort cost and the (opportunity) cost of equity.

Proposition 5. *Under perfect competition, the risk channel dominates the portfolio adjustment channel, i.e,* $\frac{dr_L}{dr} < 0$ *if and only if*

$$\frac{\partial q(r_L, r)}{\partial r}\frac{r}{q(r_L, r)} > \frac{1}{2}. \tag{15}$$

The condition on the risk elasticity for a transmission reversal is weaker than that for the monopoly case (*cf.* Equation (11)). The reason is that the monopoly bank makes a positive profit that is eroded by competition. However, the reversal rate itself can be larger or smaller under perfect competition compared to the monopoly case because risk elasticity is not the same under the two market structures. If risk elasticity is the same for the monopolistic and the competitive banks, then, ceteris paribus, the reversal rate would be unambiguously higher under perfect competition.

4.3 Liquidity shocks and endogenous deposits

In the benchmark model, Assumption 2 and the fact that the banker cannot adjust the volume of deposits implies a positive level of reserve holdings. Suppose, however, that the bank could choose the deposit volume D. In the absence of a specific reason for holding reserves, the bank would entirely abstain from holding reserves as long as the risk free rate earned on reserves was lower than the opportunity cost of deposits. Thus, the risk channel would never counteract the portfolio adjustment channel since $u'(r) \geq 0$ and a reversal of monetary policy could not occur.

Previously, we motivated the assumption of fixed deposit volume and positive reserve holdings by arguing that banks cannot easily adjust in- and outflows to their deposit accounts. In line with this motivation, we now explicitly consider idiosyncratic liquidity shocks to depositors, i.e., in- and outflows to and from their deposit accounts. For example, inflows to deposit accounts can be interpreted as sales of assets from the depositor to the central bank which automatically add to the bank's reserves holdings. Outflows from deposit accounts, in turn, need to be covered by running down reserves or by additional borrowing from the central bank. For simplicity, we assume that the bank can access the central bank's standing deposit and lending facilities at an interest rate r to deposit excess reserves or cover deposit outflows.[13] Moreover, we assume that the bank can also borrow ex ante from the central bank up to a fraction $\sigma \in (0, 1)$ of its loan issuance, i.e., we impose $R \geq -\sigma L^*$.

Liquidity shocks, denoted x, are drawn from a continuous distribution with c.d.f. $F(\cdot)$ and p.d.f. $f(\cdot)$ over support $[-1, z]$, where z denotes the maximal inflows to an individual deposit account. We assume that the liquidity shocks realize after the bank has contracted the deposit rate, set its loan rate and chosen the optimal monitoring effort. To simplify the exposition, we set $\mathbf{E}[x] = 0$ and $E = 0$.

[13] Allowing for an interest rate corridor by making the borrowing rate higher than the deposit rate would complicate this analysis without altering the main results.

As before, we solve the model backwards. Given r_D, the bank's optimal monitoring is still determined by Equation (3). The random in- and outflows to deposit accounts affect the deposit cost of the bank. In particular, if the bank is solvent, depositors receive r_D on their entire deposit holdings at maturity. With probability $1 - q$, the bank defaults. In this case, depositors obtain a pro-rata share of the remaining (positive) holdings of reserves. The expected repayment to deposits must equal their outside option $u(r)$ such that

$$r_D = \frac{u(r) - \frac{(1-\hat{q})r \int_{-\rho}^{z} (R+xD)dF(x)}{D}}{\hat{q}}. \tag{16}$$

By substituting r_D into Equation (3), we can solve the bank's monitoring choice $\hat{q}(r_L, D; r)$. The partial effects of r_L, r and D on the banker's optimal monitoring $\hat{q}$ reflect the effects of these variables on her expected profits. As before, the effects of r_L and r are ambiguous, with the respective conditions now taking into account expected deposit flows.[14] However, the effect of D on $\hat{q}$ is unambiguously negative, i.e.,$\frac{\partial \hat{q}}{\partial D} < 0$. This is because $u(r) \geq r$ so deposits are relatively more expensive than borrowing from the central bank (*cf.* Assumption 1).

Lemma 2. *The bank chooses a strictly positive loan issuance $L^*(r)$. Given Assumption 1, the bank minimizes its funding cost by choosing $R^* = -\sigma L^*$ and $D^* = (1-\sigma)L^*$.*

Lemma 2 shows that the bank borrows from the central bank on a permanent basis as long as this is feasible (i.e., if $\sigma > 0$). Even though the bank does not hold

[14] See the Appendix for details.

a positive level of reserves ex ante, the possibility that it ends up with a positive reserve balance due to random deposit inflows implies that the risk channel can still mitigate and even dominate the portfolio channel.

Proposition 6. *The risk channel dominates the portfolio adjustment channel, i.e.,* $\frac{dr_L}{dr} < 0$*, if and only if*

$$\frac{\partial q(r_L, r)}{\partial r} \frac{r}{q(r_L, r)} > \frac{1 - (1 - \hat{q})\mathbf{Prob}[x < \frac{\sigma}{1-\sigma}]}{\mathbf{Prob}[x \geq \frac{\sigma}{1-\sigma}]} > 1 \tag{17}$$

Modulo the probability of deposit outflows, the condition for the risk channel to dominate the portfolio adjustment channel is essentially the same as condition (11) when deposits are deterministic and exogenously given.[15] Condition (17) allows us to illustrate the effect of permanent central bank lending programs on the existence of the reversal rate. Consider the extreme case in which the bank could finance its entire loan portfolio by borrowing from the central bank, i.e., $\lim \sigma \to 1$. In this case, a reversal rate would cease to exist and the risk channel would never counteract the portfolio adjustment channel.[16] Such a situation is similar to the case with full deposit insurance, $\delta = 1$. The entire risk of the bank's loan issuance and the bank's exposure to interest rate risk would be borne by the central bank and lower policy rates would unambiguously increase the bank's profit.

[15] If the bank would only receive deposit inflows, then $\mathbf{Prob}[x < \frac{\sigma}{1-\sigma}] = 0$ and condition (17) would equal condition (11).

[16] The right-hand side of Equation (17) converges to ∞, while the left-hand side would assume a finite value, implying that the condition could never be satisfied.

5 Conclusion

The current environment of low and sometimes negative policy rates has given rise to concerns about the implications of low rates for the riskiness of bank portfolios and about whether low rates may impair the transmission of monetary policy. The present paper argues that these (unintended) consequences of low interest rates are two sides of the same coin as they are both caused by agency frictions between a bank and its short-term creditors.

As our model is a partial equilibrium model, it does not provide quantitative guidance about the level of the critical rate at which banks become constrained and monetary transmission weakens. However, as emphasized by Repullo (2020), partial equilibrium models can provide insights into the basic economic mechanisms and conditions that may trigger specific consequences of monetary policy, thereby providing input for larger quantitative models. In this respect, the contribution of our paper is twofold. First, we emphasize that lower policy rates that shrink bank profit margins lead banks to increase risk-taking, which in turn causes a weakening of the transmission of monetary policy to loan rates and volumes.

Second, our model admits the possibility that in an environment of sufficiently low policy rates, reductions in the policy rate can decrease banks' loan issuance. The existence of such a reversal rate depends on banks' characteristics (insured deposits, monitoring technology and the banks' capitalization) and the environment in which they operate. However, the reversal of monetary transmission is only an extreme manifestation of the more general phenomenon of weakened transmission due to higher risk-taking incentives in a low interest environment. This phenomenon should be of concern to central banks and may require them to devise policies that address the underlying causes of weaker transmission such as low capitalization, large excess liquidity or impaired deposit pass-through.

Our model suggests two policy implications that could help to alleviate the problem of weaker transmission in a low-interest rate environment. First, to counteract adverse effects on bank profitability that arise from a combination of excess liquidity and negative rates, central banks could implement reserve remuneration schemes that bolster bank profits. While such schemes redistribute seigniorage revenues back to banks, they could nevertheless strengthen the transmission and render monetary policy more effective. In this respect, our model provides a rationale for the recently introduced two-tiered remuneration for excess reserves by the Eurosystem that seeks to mitigate the effect of negative interest rates on bank profitability.[17] Second, our model suggests that the current environment of high excess liquidity and impaired deposit pass-through is conducive to the empirically observed positive relationship between bank capital and policy rates. As higher bank capital (a smaller leverage) mitigates the agency problem and lowers the reversal rate, our model echoes a recent literature (Gambacorta & Shin, 2018; Pariès et al., 2020) that argues that the capitalization of banks may matter not only for the central bank's financial stability but also for its monetary policy mandate.

[17] https://www.ecb.europa.eu/mopo/two-tier/html/index.en.html

Appendix

Proof of Lemma 1. Maximizing expected profits for a given deposit rate r_D with respect to q yields the first-order condition

$$r_L L - r_D D + rR - \kappa q = 0.$$

By substituting r_D from the participation constraint, we can obtain $\hat{q}$ as the solution to the following implicitly defined function:

$$\phi(q, r_L, r) \equiv r_L L - \frac{u(r)D - rR}{q} - \kappa q = 0.$$

The latter is quadratic in q. Following Allen et al. (2015), we take the larger of the two roots, such that

$$\frac{\partial \phi}{\partial q} = \frac{u(r)D - rR}{q^2} - \kappa < 0.$$

Moreover, we have

$$\frac{\partial \phi}{\partial r} = \frac{R - u'(r)D}{q}$$

and, using the fact that $R = D + E - L(r_L)$,

$$\frac{\partial \phi}{\partial r_L} = r_L L'(r_L) + L(r_L) - \frac{r}{q} L'(r_L).$$

An application of the implicit function theorem yields the expressions for $\partial \hat{q}/\partial r_L$ and $\partial \hat{q}/\partial r$.

□

Proof of Proposition 1. From Equation (7), the first-order condition for the optimal loan rate is given by:

$$\begin{aligned}\frac{d\Pi}{dr_L} &= \hat{q}(r_L, r)\left(r_L L'(r_L) + L(r_L)\right) - rL'(r_L) + \frac{\partial \hat{q}}{\partial r_L}\underbrace{\left(r_L L(r_L) - \kappa\hat{q}\right)}_{=(u(r)D - rR)/q} = 0 \\ &= \hat{q}(r_L, r)\left(r_L L'(r_L) + L(r_L) - \frac{r}{\hat{q}(r_L, r)}L'(r_L)\right)\left(1 - \frac{u(r)D - rR}{u(r)D - rR - \hat{q}^2\kappa}\right) = 0.\end{aligned}$$

$\hat{q}$ and the second bracket are positive, so that the optimal r_L^* satisfies condition (8) in the text.

The second-order condition, evaluated at the critical point r_L^*, becomes[18]

$$r_L L''(r_L^*) + 2L'(r_L^*) - \frac{r}{\hat{q}}L''(r_L^*) \;=\; -\frac{L''(r_L^*)L(r_L^*)}{L'(r_L^*)} + 2L'(r_L^*) \;<\; 0.$$

which is satisfied since $L(\cdot)$ is a decreasing and concave function. Thus, r_L^* maximizes the bank's profits.

Applying the implicit function theorem to the first-order condition yields:

$$\frac{dr_L^*}{dr} \;=\; -\frac{\frac{\partial \hat{q}}{\partial r}\overbrace{\left(r_L L'(r_L) + L(r_L)\right)}^{=rL'(r_L^*)/\hat{q}} - L'(r_L^*)}{,-\frac{L''(r_L^*)L(r_L^*)}{L'(r_L^*)} + 2L'(r_L^*)} \;=\; \frac{\partial r_L^*}{\partial r} + \frac{\partial r_L^*}{\partial q}\frac{\partial \hat{q}}{\partial r} \;\propto\; -L'(r_L)\left(1 - \frac{\partial \hat{q}}{\partial r}\frac{r}{\hat{q}}\right). \tag{A1}$$

Equation (A1) implies that the risk channel weakens the portfolio channel whenever $\partial\hat{q}/\partial r > 0$, which is equivalent to $u'(r) < \rho$ (cf. Lemma 1).

Next, we show the existence of a value $\bar{r}$ such that for all $r < \bar{r}$, we must have $u(r) < \rho$. Note that by Assumption 2, we must have $R = D + E - L(r_L^*(r)) > 0$ at $r = u^{-1}(\underline{u})$, while $u'(u^{-1}(\underline{u})) = 0$. Moreover if r becomes sufficiently large, R converges to a positive, but finite value, while $u'(r)$ diverges since $u(\cdot)$ is strictly convex for $r > u^{-1}(\underline{u})$. Thus, for sufficiently large r, we must have $u'(r) > \rho$, implying that there exists a smallest value $\bar{r}$ such that $u'(\bar{r}) = \rho$. For all $r < \bar{r}$, we have $u'(r) < \rho$.

Thus, for $r < \bar{r}$, we have $u'(r) < \rho$ and, as a consequence of Lemma 1, $\partial\hat{q}/\partial r > 0$. From

[18]Note that the partial effect of r_L on $\hat{q}$ is irrelevant for determining the sign of the second-order condition since $\partial\hat{q}/\partial r_L = 0$ when evaluated at r_L^*.

Equation (A1) follows immediately, that the risk channel weakens the transmission via the portfolio channel for $r < \bar{r}$. □

Proof of Proposition 2. The proof follows immediately from Equation (A1)

$$\frac{dr_L^*}{dr} < 0 \;\Leftrightarrow\; 1 < \frac{\partial \hat{q}}{\partial r}\frac{r}{\hat{q}}.$$

□

Proof of Proposition 3. We show the existence of $\hat{r}$ that satisfies

$$1 = \frac{\partial \hat{q}(r_L^*, \hat{r})}{\partial r}\frac{\hat{r}}{\hat{q}(r_L^*, \hat{r})}.$$

From the proof of Lemma 1 follows that

$$\frac{\partial \hat{q}}{\partial r}\frac{r}{\hat{q}} \;=\; \frac{(u'(r)D - R)r}{u(r)D - rR - \hat{q}^2\kappa} \;\geq\; 1 \;\Leftrightarrow\; (u(r) - u'(r)r)D \;\geq\; \kappa\hat{q}^2.$$

The last inequality cannot be satisfied as long as $u'(r)D \geq R$, since $u(r)D - rR < \kappa\hat{q}^2$ (cf. Lemma 1). Therefore, we restrict attention to $u'(r) < \rho$. Since $u''(r) > 0$, the left-hand side of the above inequality is strictly decreasing in r, implying that $\text{argmax}_r\{u(r) - u'(r)r\} = u^{-1}(\underline{u})$. Thus, as long as $\kappa\hat{q}^2 > \underline{u}$, the risk channel can never dominate the portfolio adjustment channel. Hence, a necessary and sufficient condition for the existence of a reversal rate is that κ satisfies $\kappa\hat{q}^2 \leq \underline{u}$. Note further that

$$\frac{d\kappa\hat{q}(\kappa)^2}{d\kappa} \;=\; \frac{\hat{q}^2}{u(r)D - rR - \hat{q}^2\kappa}\left(u(r)D - rR + \kappa\hat{q}^2\right) \;<\; 0.$$

Thus, we can find a value $\underline{\kappa}$ such that $\underline{\kappa}\hat{q}(\underline{\kappa})^2 = \underline{u}$ where $\underline{\kappa}$ also satisfies the condition for $\partial\phi/\partial q < 0$ in the proof of Lemma 1. Since $u(r) - u'(r)\hat{r})D - \kappa\hat{q}^2$ is strictly decreasing in r, there exists $\hat{r}$ such that for $\kappa \geq \underline{\kappa}$

$$(u(\hat{r}) - u'(\hat{r})\hat{r})D - \kappa\hat{q}^2 = 0. \tag{A2}$$

For $r < \hat{r}$, we have

$$(u(r) - u'(r)r)D > \kappa\hat{q}^2 \;\Leftrightarrow\; \frac{\partial\hat{q}}{\partial r}\frac{r}{\hat{q}} > 1.$$

□

Proof Hypothesis 1. We consider an exogenous increase in the deposit volume for a given E and show that this leads to an increase in excess reserves, a higher reserve-deposit ratio and less lending. The equilibrium effect of an increased D follows by applying the implicit function theorem to the two equilibrium conditions

$$r_L L(r_L) - \frac{u(r)D - r(D + E - L(r_L))}{q} - \kappa q = 0,$$
$$r_L L'(r_L) + L(r_L) - \frac{rL'(r_L)}{q} = 0.$$

Let $\mathbf{J}^*$ denote the Jacobian of the above system of two equations evaluated at the optimum. From the proofs of Lemma 1 and Proposition 1 follows that $\mathbf{J}^* < 0$ (when the variable vector is (r_L, q)). Note further that the second equation is independent of D. Thus, by the implicit function theorem

$$\frac{dq^*}{dD} \propto -\frac{u(r) - r}{q^*} < 0 \quad \text{and} \quad \frac{dr_L^*}{dD} \propto \frac{u(r) - r}{q^*} > 0.$$

Since r_L increases in D, a higher D leads to less lending and higher excess reserves

$$\frac{dL^*}{dD} = L'(r_L^*)\frac{dr_L^*}{dD} < 0 \quad \text{and} \quad dR = dD - L'(r_L^*)\frac{dr_L^*}{dD} > 0.$$

Finally, note that the latter implies also a higher reserves-deposit ratio ρ because $\rho < 1$ and $L'(r_L^*)\frac{dr_L^*}{dD} < 0$ such that we obtain

$$\frac{d\rho}{dD} = \frac{1}{D}\left(1 - \rho - L'(r_L^*)\frac{dr_L^*}{dD})\right) > 0.$$

□

Proof Hypothesis 2. Applying the implicit function theorem to Equation (A2) yields:

$$\frac{\partial \hat{r}}{\partial \kappa} = \frac{\frac{d\kappa \hat{q}^2}{d\kappa}}{-ru''(r)D - 2\kappa\hat{q}\frac{\partial \hat{q}}{\partial r}} > 0 \quad \text{and} \quad \frac{\partial \hat{r}}{\partial E} = \frac{2\hat{q}\kappa\frac{\partial \hat{q}}{\partial E}}{-ru''(r)D - 2\kappa\hat{q}\frac{\partial \hat{q}}{\partial r}} < 0.$$

□

Proof of Proposition 4. $\hat{q}$ is given as the solution to the following implicit function:

$$\phi(q, r_L, \delta, r) \equiv r_L L(r_L) - \left(\delta + \frac{1-\delta}{q}\right)(u(r)D - rR) - \kappa q = 0,$$

with

$$\frac{\partial \phi}{\partial q} = \frac{1-\delta}{q^2}(u(r)D - rR) - \kappa < 0,$$

$$\frac{\partial \phi}{\partial r} = \frac{(R - u(r)D)(q\delta + (1-\delta))}{q^2} > 0 \Leftrightarrow \rho > u'(r),$$

$$\frac{\partial \phi}{\partial r_L} = r_L L'(r_L) + L(r_L) - \frac{\delta q + (1-\delta}{q} r L'(r_L),$$

and

$$\frac{\partial \phi}{\partial \delta} = \frac{1-q}{q}(u(r)D - rR) > 0.$$

Given $\hat{q}$, the first-order condition for the banker's optimal loan rate is given by

$$\hat{q}\left(r_L L'(r_L) + L(r_L) - \frac{\delta q + (1-\delta}{q} r L'(r_L)\right)\left(1 - \frac{(\hat{q}\delta + (1-\delta))(u(r)D - rR)}{(1-\delta)(u(r)D - rR) - \kappa\hat{q}^2}\right) = 0.$$

Since the second bracket is strictly positive, the optimal loan rate satisfies

$$r_L L'(r_L) + L(r_L) - \frac{\delta q + (1-\delta}{q} r L'(r_L) = 0.$$

Application of the implicit function theorem yields

$$\frac{dr_L^*}{dr} \propto -(1-\delta)L'(r_L)\left(1 + \frac{\delta\hat{q}}{1-\delta} - \frac{\partial \hat{q}}{\partial r}\frac{r}{\hat{q}}\right).$$

Thus, $\frac{dr_L^*}{dr} < 0$ if and only if $1 + \frac{\delta \hat{q}}{1-\delta} < \frac{\partial \hat{q}}{\partial r}\frac{r}{\hat{q}}$. □

Proof of Hypothesis 3. From the proof of Proposition 4 follows that the reversal rate $\hat{r}(\delta)$ is given by the solution to

$$\frac{\partial \hat{q}}{\partial r}\frac{r}{\hat{q}} - 1 - \frac{\delta \hat{q}}{1-\delta} = 0.$$

Using the expressions for $\partial \hat{q}/\partial r$, we can rewrite the latter as

$$u(r)D - \delta r R - (1-\delta)u'(r)rD - \kappa \hat{q}^2 = 0. \tag{A3}$$

For $\delta = 0$, the above condition is equal to Equation (A2), implying that $\hat{r}(\delta)$ converges to the value of the reversal rate in Proposition 3. Another application of the implicit function theorem to Equation (A3), taking into account that for $r = \hat{r}$ we have $u'(r) < \rho$ and $\partial R/\partial r = 0$, implies $\frac{\partial \hat{r}}{\partial \delta} < 0$.

Note further that for $\delta \to 1$, Equation (A3) cannot be satisfied since $\hat{q}$ is the larger root, which implies that $\kappa \hat{q}^2 - u(r)D + rR > 0$. Hence, for $\delta \to 1$, the reversal rate ceases to exist.

□

Proof of Proposition 5. Given a loan market equilibrium, the monitoring effort is implicitly defined by the same condition as before, which we obtain by combining Equation (3) and Equation (4). In particular, we obtain

$$\frac{\partial \hat{q}}{\partial r} = -\frac{R - u^{'}D}{\frac{u(r)D - rR}{\hat{q}} - \kappa \hat{q}.} \tag{A4}$$

Note that for an interior solution to exist, the profits must be concave in $\hat{q}$, i.e., $\frac{u(r)D-rR}{\hat{q}} - \kappa \hat{q} < 0$. Equation (12) and Equation (13) simultaneously define the perfectly competitive banking sector equilibrium.

Substituting Equation (12) into Equation (13) we obtain Equation (14).

Total differentiation of Equation (14) gives

$$\frac{d\,r_L}{d\,r} = -\frac{-E + \kappa \hat{q}(r_L, r)\frac{\partial \hat{q}}{\partial r}}{\kappa \hat{q}(r_L, r)\frac{\partial \hat{q}}{\partial r_L}..} \tag{A5}$$

The denominator is positive because in a perfectly competitive equilibrium it must hold that $\frac{\partial \Pi}{\partial r_L} > 0$; otherwise, a bank could profitably deviate by decreasing the loan rate. The sign of loan rate transmission is dictated by $E - \kappa \hat{q}(r_L, r)\frac{\partial \hat{q}}{\partial r}$.

Expanding by $\frac{r}{\hat{q}}$ and using Equation (14), the condition for $\frac{d\,r_L}{d\,r} < 0$ becomes

$$\frac{r}{q}\frac{\partial \hat{q}}{\partial r} > \frac{1}{2}. \tag{A6}$$

□

Proof of Lemma 2 and Proposition 6. The adjusted profit function becomes

$$\Pi = q\left(r_L L(r_L) + r\int_{-1}^{z}(R + x\,D)\,dF(x) - r_D D\right) - \frac{\kappa q^2}{2}.$$

Because $\mathbf{E}[x] = 0$, we can simplify to the same profit function as in our baseline model

$$\Pi = q\left(r_L L(r_L) + r\,R - r_D D\right) - \frac{\kappa q^2}{2}.$$

Inserting the participation constraint into the first order condition for q implicitly defines the function $\hat{q}(r_L, D, r)$

$$\phi(r_L, D, r) = r_L L(r_L) + r\,R - \frac{u(r)D - (1-q)r\int_{-1}^{-\rho}(R + xD)\,dF(x)}{q} - \kappa q = 0.$$

Taking the larger of the two roots, we obtain

$$\frac{\partial \phi}{\partial r} = qR - u^{'}D + (1-q)\int_{-\rho}^{z}(R + xD)\,dF(x) = R - (1-q)\int_{-1}^{-\rho}(R + xD)\,dF(x) - u^{'}D$$

and

$$\frac{\partial \phi}{\partial r} = q((r_L - r)L^{'} + L) - (1-q)rL^{'}\int_{-\rho}^{z}dF(x) = q(r_L L^{'} + L) - rL^{'} + (1-q)rL^{'}\int_{-1}^{-\rho}dF(x)$$

as well as

$$\frac{\partial \phi}{\partial D} = qr - u(r) + (1-q)r \int_{-\rho}^{z} (1+x)\, dF(x) = r - u(r) - (1-q)r \int_{-1}^{-\rho} (1+x)\, dF(x) < 0$$

which is unambiguously negative for $r \leq u(r)$.

The first-stage profit function, given the required return for the expected equilibrium monitoring choice becomes

$$\Pi(r_L, D; r) = \hat{q}(r_L L(r_L) + r\, R) - u(r)D - (1-q)r \int_{-1}^{-\rho} (R + xD)\, dF(x) - \kappa \frac{\hat{q}^2}{2}.$$

Differentiating with respect to D and r_L yields the first-order conditions for a profit maximum. As the bank optimally minimizes deposit costs, we evaluate the first order condition at $D^* = (1-\sigma)L^*(r_L)$ and $R^* = -\sigma L^*(r_L)$, such that $\rho = -\frac{\sigma}{1-\sigma}$.

Using the implicit function theorem we then obtain

$$\frac{dr_L}{dr} = -\frac{-L' + (1-\hat{q})L' \int_{-1}^{\frac{\sigma}{1-\sigma}}\ dF(x)) + \left(r_L L' + L - rL' \int_{-1}^{\frac{\sigma}{1-\sigma}}\ dF(x)\right) \frac{\partial \hat{q}}{\partial r}}{\frac{\partial^2 \Pi}{\partial r_L^2}}.$$

For r_L^* to be the optimal loan rate in equilibrium, it must hold that $\frac{\partial^2 \Pi}{\partial r_L^2} < 0$. Therefore, $\frac{dr_L}{dr} < 0$ if and only if the numerator is negative. Using the first-order condition $\frac{\partial \Pi}{\partial r_L} = 0$, we can simplify to

$$\frac{r}{\hat{q}} \left(1 - \int_{-1}^{\frac{\sigma}{1-\sigma}}\ dF(x)\right) \frac{\partial \hat{q}}{\partial r} > 1 - (1-\hat{q}) \int_{-1}^{\frac{\sigma}{1-\sigma}}\ dF(x).$$

Note that $\lim \sigma \to 1$, the left hand side approaches zero and the right hand side $\hat{q}$ such that the condition can never be fulfilled. If the bank can fund all loan assets paying r for borrowing from the central bank, no reversal rate can exist. □

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