The Risk-Return Trade-Off Among Equity Factors

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Abstract

We examine the time-series risk-return trade-off among equity factors. We obtain a positive trade-off for profitability and investment factors. Such relationship subsists conditional on the covariance with the market factor, which represents consistency with Merton's ICAPM. Critically, we obtain an insignificant risk-return relationship for the market factor. The factor risk-return trade-off tends to be weaker among international equity markets. The out-of-sample forecasting power (of factor variances for future own returns) tends to be economically significant for the investment and profitability factors. Our results suggest that the risk-return trade-off is stronger within segments of the stock market than for the whole.

Keywords: Asset pricing, risk-return trade-off, ICAPM, realized volatility, profitability and investment factors

JEL classification: G11; G12; G17

1 Introduction

According to conditional versions of the CAPM of Sharpe (1964) and Lintner (1965), there should be a positive aggregate risk-return trade-off, that is, a positive association between the (conditional) variance of the market return and its (conditional) expected return. This simple prediction has been the focus of an extensive empirical literature. In particular, by using different empirical methods several studies have shown that such positive risk-return relation exists at the aggregate level (e.g., Bollerslev, Engle, and Wooldridge 1988, Scruggs 1998, Harrison and Zhang 1999, Ghysels, Santa-Clara, and Valkanov 2005, Guo and Whitelaw 2006, Lundblad 2007, Pástor, Sinha, and Swaminathan 2008, Bali and Engle 2010, and Hedegaard and Hodrick 2016). On the other hand, several studies find a negative aggregate risk-return relation (e.g., Campbell 1987, Nelson 1991, Glosten, Jagannathan, and Runkle 1993, Whitelaw 1994, Brandt and Kang 2004, among others).[1] However, most of this literature has focused on examining the market risk-return trade-off, with few studies assessing this trade-off for components of the stock market.[2] We contribute to filling this gap in the literature by examining the risk-return trade-off among some of the most popular zero-cost equity factors in the asset pricing literature.

In the same vein that the market risk-return trade-off is consistent with the conditional CAPM, the factor risk-return relation is consistent with conditional two-factor models that can be interpreted as empirical applications of the ICAPM of Merton (1973). Specifically, we employ conditional two-factor models containing the market and each of the non-market factors employed in the multifactor models of Carhart (1997) and Fama and French (2015, 2016) to motivate and explain our empirical tests. According to those models, there should exist a positive relation between the risk premium and conditional variance of the size, value, momentum, profitability, and investment risk factors, controlling by the conditional covariance with the market factor. To proxy for the unobserved conditional factor variances and covariances, we compute monthly realized factor variances and covariances based on daily factor observations, an approach that is widely adopted in

[1] See also Merton (1980), Turner, Startz, and Nelson (1989), Glosten, Jagannathan, and Runkle (1993), and Lettau and Ludvigson (2010). In related work, Yu and Yuan (2011) and Guo, Wang, and Yang (2013) show that the risk-return relation changes over time.

[2] One exception is the work of Bali (2008), who looks at the effect of conditional (equity portfolio) covariances with the market factor onto portfolio's expected returns. Guo, Savickas, Wang, and Yang (2009) forecast the value premium based on the conditional covariance with the market and the own conditional variance. On the other hand, Wang, Yan, and Yu (2017) examine the risk-return trade-off among individual stocks.

the risk-trade-off literature (e.g., Haugen, Talmor, and Torous 1991, Goyal and Santa-Clara 2003, Bali, Cakici, Yan, and Zhang 2005, Wei and Zhang 2005, Guo 2006, Guo and Savickas 2006, Guo, Savickas, Wang, and Yang 2009, Barroso and Santa-Clara 2015, Lof 2019, among others).

By using monthly data from 1967 to 2016, the results of univariate predictive regressions suggest a positive in-sample risk-return trade-off for the profitability (RMW) and investment (CMA) factors of Fama and French (2015). This positive risk-return trade-off is economically significant as indicated by the magnitudes of the slopes associated with lagged factor variance. On the other hand, we do not find a significant positive risk-return trade-off for the remaining equity factors, including the market factor.

By estimating bivariate predictive regressions, which include the realized covariance with the market factor, we obtain a good empirical consistency with the two-factor ICAPM when it comes to forecasting either RMW or CMA: In both cases, we obtain significant positive slope estimates associated with the realized factor variances, in addition to obtaining plausible estimates of the risk aversion parameter. It turns out that the realized covariances (with the market factor) play a secondary role, in comparison to the own factor variances, as indicated by the weaker statistical significance (associated with the first group of realized moments). However, it is the case that these covariance terms clarify the role of the own variances when it comes to predicting the future monthly returns of these factors.

We conduct several robustness checks. Our results suggest that the positive risk-return trade-off associated with both RMW and CMA is more relevant during periods of economic slowdown. Second, the positive risk-return association for those two factors remains strongly significant by using an alternative statistical inference based on a bootstrap simulation. Third, we find that the positive risk-return trade-off associated with the investment factor remains significant over a shorter sample that ends in 2007, while in the case of RMW we obtain weaker significance than for the full sample. This suggests that the risk-return trade-off associated with the profitability factor became more important in recent years.

Overall, our results indicate that a positive risk-return trade-off is much more pervasive for specific segments of the U.S. stock market (e.g., profitability and investment sorted portfolios) than for the market as a whole. To put these results in perspective, we examine the factor risk-return trade-off in international stock markets over a shorter period (1990 to 2016). The results suggest that the positive risk-return trade-off associated with the profitability and investment factors is less robust among international equity markets. Still, there is some evidence that such trade-off exists in the North-American and European markets. On the other hand, the covariance with the market factor does not help predicting factor risk premia in most cases.

In the last part of the paper, we examine the economic significance of the out-of-sample forecasts associated with factor realized variances for future factor returns. To achieve that goal, we construct a trading strategy that relies on such predictability. Specifically, the strategy times each factor by going long (short) the factor whenever the predicted factor risk premium (obtained from the predictive regressions in recursive samples) is positive (negative). This factor exposure comes in addition to a permanent long position in the stock market index, and this dynamic strategy is compared against a simple "buy-hold" strategy on the stock index.

The results indicate that the out-of-sample forecasting power (of realized volatility for future factor returns) is economically significant in the cases of the profitability and investment factors. Specifically, the annual pseudo Sharpe ratios are above one in both cases compared to 0.69 for the passive strategy that only invests in the stock market index. Moreover, the utility gains associated with the dynamic strategies corresponding to those two factors are above 3.5% per year in most cases. In comparison, such economic significance does not exist or is not robust for the other equity factors. These results are robust to using different formulations of the trading strategy, namely employing different factor weights, using a different in-sample period to obtain the forecasts, constraining the slope estimates for being positive, or employing bivariate predictive regressions (that contain the realized covariances with the market factor). By using an alternative trading strategy, which explores only positive factor risk premia, we obtain economically significant gains in the case of the profitability factor.

The analysis conducted in this study allows us to select "proper" non-market risk factors, that is, factors with positive risk-return tradeoff after controlling for the market factor. This can be interpreted as another method of selecting equity risk factors, which (combined with the market factor) originate better specified (or more theoretically sounded) multifactor models (to price cross-sectional risk premia). For example, the results in the paper suggest that the RMW and CMA seem valid risk factors to include in a multifactor model, but that does not seem to be the case with the size and momentum factors. Therefore, this study provides some discipline on the selection of traded equity factors included in multifactor models.

This study is directly related to the large literature on the time-series risk-return trade-off summarized above. Our work is more directly related with a few studies that analyze the risk-return trade-off associated with non-market equity factors (e.g., Charoenrook and Conrad 2008 and Guo, Savickas, Wang, and Yang 2009). Among other aspects, we differentiate from these two studies by focusing on the recent investment and profitability factors proposed by Fama and French (2015, 2016), which are absent from their papers. In a related study, Huang and Wang (2014) investigate the risk-return trade-off for an earlier version of the investment and profitability factors employed in Hou, Xue, and Zhang (2015). We differentiate from Huang and Wang (2014) in three major aspects. First, we rely on realized second moments (as a proxy for the unobserved conditional moments)—the most popular method employed in this literature—instead of estimating a GARCH-M model. Second, our empirical analysis centers on the profitability and investment factors of Fama and French (2015, 2016). There is strong evidence that the empirical performance of these two factors differs substantially from that of the corresponding profitability and investment factors of Hou, Xue, and Zhang (2015) (see Maio and Santa-Clara 2017, Maio and Philip 2018, Cooper and Maio 2019b, Hou, Mo, Xue, and Zhang 2019, 2021, Cooper, Ma, and Maio 2022, among others). Third, we assess the out-of-sample time-series risk-return trade-off for each equity factor, something that is missing in Huang and Wang (2014).

This paper also contributes on the growing literature that studies the asset pricing implications of the new multifactor models, with particular focus on the profitability and investment factors. Specifically, our work is indirectly related with those studies that look at the asset pricing implications of the mentioned factors without relying exclusively on cross-sectional tests of multifactor models (e.g., Barillas and Shanken 2017, 2018, Fama and French 2018, Hou, Mo, Xue, and Zhang 2019). These studies employ spanning regressions or compute statistics based on Sharpe ratios to select the "best" traded risk factors. In contrast, we employ the consistency with ICAPM's implications for the time-series risk-return tradeoff to select the "best" (or more theoretically sounded) traded factors. Hence, while the approach used in those studies is purely empirical, our approach involves placing restrictions on the baseline stochastic discount factor that prices assets.

The paper proceeds as follows. Section 2 provides the theoretical foundations for the factor risk-return trade-off, while Section 3 describes the data and variables. In Section 4, we estimate the risk-return relation among equity factors. Section 5 provides evidence for the factor risk-return trade-off in international markets. In Section 6, we examine the out-of-sample predictability of factor variances for factor risk premia. Finally, Section 7 concludes.

2 Background

In this section, we provide the theoretical foundation for the empirical analysis conducted in the following sections.

Consider a two-factor linear asset pricing model in which the factors are the excess market return (RM) and the return on a zero-cost equity portfolio (F), which also represents an excess return. This model can be interpreted as an empirical application of the Intertemporal CAPM (ICAPM) of Merton (1973). In this setup, F represents the "hedging" risk factor, that is, the factor that hedges for future changes in the investment opportunity set (see Vassalou 2003, Maio and Santa-Clara 2012, and Cooper and Maio 2019a, among others).[3] Maio and Santa-Clara (2012) and Cooper and Maio (2019a) show that many of the equity factors analyzed in the following sections satisfy a sign restriction with the ICAPM. This means that state variables associated with those factors forecast future aggregate investment opportunities (stock market return, stock market volatility, and/or economic activity) in a way that is compatible with the corresponding positive risk prices.[4] Hence, this legitimates the two-factor models analyzed below as empirical applications of the ICAPM.

The stochastic discount factor (SDF) representation of this two-factor model is given by

$$0 = \mathrm{E}_t(M_{t+1} R^e_{i,t+1}), \tag{1}$$

$$M_{t+1} = 1 + b_{RM} RM_{t+1} + b_F F_{t+1}. \tag{2}$$

In this representation, M denotes the SDF that prices assets; R^e_i represents the excess return on a risky asset i; and $\mathrm{E}_t(\cdot)$ represents the conditional expectation at time t.[5] We are assuming for

[3] Another possible explanation is that this two-factor model is consistent with the APT framework of Ross (1976) to the extent that these two traded factors explain a large share of the time-series variation in the cross-section of stock returns (e.g., Ahn, Horenstein, and Wang 2018 and Cooper, Ma, Maio, and Philip 2021).

[4] Specifically, Cooper and Maio (2019a) show that investment factors forecast a decline in aggregate volatility and an improvement in future economic activity. On the other hand, profitability factors forecast an increase in the aggregate equity premium. In related work, Barroso, Boons, and Karehnke (2021) test the consistency with the ICAPM of linear combinations of the equity factors.

[5] The excess return can represent either the difference between a risky return and the risk-free rate or the spread

simplicity that the SDF coefficients (b_j) are time invariant.[6]

It is well known that the above representation is equivalent to the following expected return-covariance equation,

$$\mathrm{E}_t(R^e_{i,t+1}) = -b_{RM}\,\mathrm{cov}_t(R^e_{i,t+1}, RM_{t+1}) - b_F\,\mathrm{cov}_t(R^e_{i,t+1}, F_{t+1}), \tag{3}$$

where $\mathrm{cov}_t(R^e_{i,t+1}, \cdot)$ denotes the conditional covariance at time t between the excess return and each of the factors.[7]

Since the two factors are traded, the pricing equation above also applies to each factor in the SDF (e.g., Cochrane 2005, Lewellen, Nagel, and Shanken 2010, and Peñaranda and Sentana 2015),

$$\mathrm{E}_t(RM_{t+1}) = -b_{RM}\,\mathrm{var}_t(RM_{t+1}) - b_F\,\mathrm{cov}_t(RM_{t+1}, F_{t+1}), \tag{4}$$

$$\mathrm{E}_t(F_{t+1}) = -b_{RM}\,\mathrm{cov}_t(F_{t+1}, RM_{t+1}) - b_F\,\mathrm{var}_t(F_{t+1}), \tag{5}$$

where $\mathrm{var}_t(\cdot)$ denotes the conditional variance at time t.

These equations illustrate the direct risk-return trade-off for each factor: if $b_j < 0, j = RM, F$ (that is, a positive innovation in each factor translates into a lower realization of the SDF, that is, lower growth in the marginal value of wealth), we have that an increase in risk (as measured by a rise in the factor's conditional variance) translates into higher conditional factor risk premia. Moreover, an increase in the conditional covariance between the two factors also translates into a a higher factor risk premium. In the context of the ICAPM, $-b_{RM}$ represents the average relative risk aversion in the economy. $b_F < 0$ means that a positive shock in the hedging factor signals

between two risky returns.

[6]Most empirical tests of asset pricing models make this assumption. This is especially the case in the time-series aggregate risk-return trade-off literature, with which this study relates the most.

[7]This equation is widely used in the literature, but represents an approximation of the true pricing equation. However, this approximation is accurate to a very large degree. Given the usual assumption of a gross risk-free rate (denoted by $R_{f,t+1}$), we have $\mathrm{E}_t(M_{t+1}) = 1/R_{f,t+1}$. This implies that the pricing equation is defined as

$$\mathrm{E}_t(R^e_{i,t+1}) = -b_{RM} R_{f,t+1}\,\mathrm{cov}_t(R^e_{i,t+1}, RM_{t+1}) - b_F R_{f,t+1}\,\mathrm{cov}_t(R^e_{i,t+1}, F_{t+1}).$$

In practical terms, $R_{f,t+1}$ is a number very close to one, which implies $\mathrm{E}_t(M_{t+1}) = R^{-1}_{f,t+1} \approx 1$, leading to

$$\mathrm{E}_t(R^e_{i,t+1}) \approx -b_{RM}\,\mathrm{cov}_t(R^e_{i,t+1}, RM_{t+1}) - b_F\,\mathrm{cov}_t(R^e_{i,t+1}, F_{t+1}).$$

an improvement in future investment opportunities (that is, good times), which implies a negative shock to the marginal value of multiperiod wealth (i.e., M).

Assuming that we have an empirical proxy for the unobserved conditional variance of each factor, as well as for the conditional covariance between the two factors, the risk-return trade-off can be tested empirically through the following predictive regressions,

$$RM_{t+1} = \alpha_{RM} + \theta_{RM} \operatorname{var}_t(RM_{t+1}) + \theta_{RM,F} \operatorname{cov}_t(RM_{t+1}, F_{t+1}) + \varepsilon_{RM,t+1}, \tag{6}$$

$$F_{t+1} = \alpha_F + \theta_{F,RM} \operatorname{cov}_t(RM_{t+1}, F_{t+1}) + \theta_F \operatorname{var}_t(F_{t+1}) + \varepsilon_{F,t+1}, \tag{7}$$

where ε_{RM} and ε_F represent zero-mean forecasting errors. By taking conditional expectations, we obtain:

$$\mathrm{E}_t(RM_{t+1}) = \alpha_{RM} + \theta_{RM} \operatorname{var}_t(RM_{t+1}) + \theta_{RM,F} \operatorname{cov}_t(RM_{t+1}, F_{t+1}), \tag{8}$$

$$\mathrm{E}_t(F_{t+1}) = \alpha_F + \theta_{F,RM} \operatorname{cov}_t(RM_{t+1}, F_{t+1}) + \theta_F \operatorname{var}_t(F_{t+1}). \tag{9}$$

These two equations make clear that the intercept estimates in the time-series regressions (α_{RM}, α_F) represent estimates of the pricing errors for each testing portfolio (or factor). If the two-factor ICAPM is perfectly specified (zero pricing errors), we should obtain intercept estimates that are not statistically different from zero. However, it is likely that the two-factor model is misspecified, that is, there are more risk factors in the true pricing kernel that prices equity portfolios. This implies non-zero intercepts for at least some of the traded portfolios (or factors) being priced. On the other hand, restricting the intercepts to zero, might originate a more powerful test of the time-series risk-return trade-off: By forcing zero pricing errors, we want to check if there is still a positive relationship between risk (captured by the variance and covariance terms) and the expected return of equity portfolios (or factors).

Another point deserves attention regarding the bivariate regressions above. Given the relationships, $\theta_{RM} \equiv -b_{RM}, \theta_{RM,F} \equiv -b_F, \theta_{F,RM} \equiv -b_{RM}, \theta_F \equiv -b_F$, there is a sign restriction on the slopes from the predictive regressions: Under the assumption that the two SDF coefficients are negative, all the slopes should be estimated positively. This is consistent with the factors having a positive risk premium, which holds for all equity factors analyzed in the following sections.

If we further assume that the own conditional variance is significantly more important in driving factor risk premia than the conditional covariance with the other factor, we have the following approximation to the conditional model stated above,

$$\mathrm{E}_t(RM_{t+1}) \approx -b_{RM}\,\mathrm{var}_t(RM_{t+1}), \quad (10)$$

$$\mathrm{E}_t(F_{t+1}) \approx -b_F\,\mathrm{var}_t(F_{t+1}), \quad (11)$$

which can be estimated from the following univariate regressions:

$$RM_{t+1} = \alpha_{RM} + \theta_{RM}\,\mathrm{var}_t(RM_{t+1}) + \varepsilon_{RM,t+1}, \quad (12)$$

$$F_{t+1} = \alpha_F + \theta_F\,\mathrm{var}_t(F_{t+1}) + \varepsilon_{F,t+1}. \quad (13)$$

The equation for the market factor is exact, rather than an approximation, under a simple conditional version (e.g., using conditional moments) of the CAPM of Sharpe (1964) and Lintner (1965). In the regression for the market factor, $\theta_{RM} \equiv -b_{RM}$ represents an estimate of the risk aversion coefficient. This relation has been the focus of the large empirical risk-return trade-off literature.[8]

In the following sections, F represents one of the (zero-cost) size (SMB), value (HML), momentum (UMD), profitability (RMW), and investment (CMA) factors. Hence, each of these two-factor models represent a restricted version of the multifactor models of Fama and French (1993, 2015, 2016) and Carhart (1997).[9] A priori, the long-short equity portfolio associated with any of the anomalies covered in Hou, Xue, and Zhang (2020) represents a plausible empirical proxy for F. However, we restrict our analysis to traded factors, which are used in the literature as "risk factors" that price cross-sectional equity risk premia.

[8]This equation is also compatible with the ICAPM of Merton (1973) if we assume that the average investor has log utility and/or investment opportunities are time-invariant (and hence, the "hedging" factors are not priced in equilibrium).

[9]The size factor in the five-factor model of Fama and French (2015, 2016) is constructed in a slightly different way than the original SMB of Fama and French (1993) (see Fama and French 2015 for details). To save space and keep the focus, we only provide results for the original size factor. The results for the alternative size factor are quite similar and are available upon request from the authors.

More concretely, we estimate the following forecasting regressions for the momentum factor,[10]

$$UMD_{t+1} = \alpha_{UMD} + \theta_{UMD,RM} \operatorname{cov}_t(RM_{t+1}, UMD_{t+1}) + \theta_{UMD} \operatorname{var}_t(UMD_{t+1}) + \varepsilon_{UMD,t+1}, \quad (14)$$

$$UMD_{t+1} = \alpha_{UMD} + \theta_{UMD} \operatorname{var}_t(UMD_{t+1}) + \varepsilon_{UMD,t+1}, \quad (15)$$

and similarly for each of the other factors enumerated above.

The univariate predictive regression represents only an approximation to the two-factor model since it ignores the conditional covariance (between the momentum factor and the market return). Hence, the larger is the term related with the realized covariance above, the less valid becomes the univariate regression. The estimate of $\theta_{UMD,RM}$ in the bivariate regression represents an estimate of the risk aversion parameter, as explained above.

We note that the forecasting regressions stated above represent an empirical test of the two-factor ICAPM. This means that in each regression there is only one non-market factor (e.g., SMB). Thus, we do not attempt to test a conditional version of the original equity multifactor models (e.g., the four-factor model of Carhart (1997)), which were designed to price cross-sectional equity risk premia. Instead, our aim is to provide a test for the time-series risk-return trade-off associated with several equity factors existent in the literature within the simplest ICAPM framework (a model containing the market factor and one of the long-short equity factors). Indeed, the generic goal in this paper is to evaluate whether the time-series risk-return tradeoff is stronger within segments of the stock market than the whole, which makes the aggregate risk-return tradeoff as our reference point. Since the market risk-return tradedoff is consistent with a conditional CAPM, as shown above, it makes sense in our case to use a model that is the closest to the baseline conditional CAPM. This the case of the two-factor conditional ICAPM (rather than a higher-order ICAPM).

All the equity factors described above are designed in a way to deliver positive risk premiums. Hence, we expect that the estimated own slopes from the predictive regressions are positive in all cases. This is also consistent with the hypothesis stated above that each of these factors are positively correlated with future aggregate investment opportunities (i.e., positive realizations in the factors signal good times), and hence negatively correlated with the SDF associated with the

[10] A similar two-factor forecasting model is estimated in Guo, Savickas, Wang, and Yang (2009) for the case of the value factor.

ICAPM (i.e., negative SDF coefficients). In the following sections, we aim to test empirically these risk-return relations for each of the equity factors.

3 Data and Variables

In this section, we describe the data and variables employed in the following sections.

To estimate the risk-return trade-off for each equity factor we need an empirical proxy for the unobserved conditional factor variances. Following a large bulk of the literature (e.g., Haugen, Talmor, and Torous 1991, Goyal and Santa-Clara 2003, Bali, Cakici, Yan, and Zhang 2005, Guo 2006, Guo and Savickas 2006, Barroso, Detzel, and Maio 2021, among others), we use the realized variance in month t,

$$RV_t = \sum_{n=0}^{20} F_{d_t-n}^2, \tag{16}$$

as an estimate of the conditional variance of a given factor F, $(\text{var}_t(F_{t+1}))$. In the above expression, F_{d_t-n} denotes each of the last 21 daily realizations of F and $d_t, t = 1, ..., T$ represents the time-series of the dates of the last trading sessions of each month. The data on the daily factors are obtained from Kenneth French's data library. The sample is 1967:02 to 2016:12.

Table 1 presents the descriptive statistics for the realized variances. The realized variance of UMD is by far the most volatile (standard deviation above 1% per month), followed by the realized market variance (0.41%). The other factors exhibit significantly less volatility of realized variance (around or lower than 0.10%). The plots of the time-series of the realized variances, which are presented in the online appendix, indicate that the most remarkable spikes in market variance are centered around the 1987 stock market crash and the more recent (2007–09) financial crisis, whereas the size premium variance also exhibited a major rise in the 1987 crash. On the other hand, the realized variances of the profitability and investment factors show a sharp increase around the correction of the NASDAQ bubble (in the early 2000s). Regarding the momentum and value factors, the largest spike in their realized variances occurs in the recent crisis. These results suggest that several of the factor realized variances are only weakly correlated.

Table 2 shows the estimates of an AR(1) process for each factor realized variance. The results show that, apart from the size factor, all equity factors have realized variances that are somewhat persistent over time, as indicated by the autoregressive coefficients on the 0.71-0.77 range (which are strongly significant). The corresponding R^2 estimates are above 50% in all cases. In comparison, the market variance is significantly less persistent than these other factors with an AR(1) slope of 0.53.

The data on the monthly equity factors are also obtained from Kenneth French's web page. To save space, the corresponding descriptive statistics are presented in the appendix. The factor with the largest mean is clearly UMD (0.66% per month), followed by the aggregate equity premium (0.51%). On the other hand, the factor with the lowest mean return is SMB (0.20% per month), followed by RMW (0.26%). The most volatile factors are the aggregate equity premium and the momentum factor, with standard deviations above 4% per month. At the other end of the spectrum, the investment factor is the least volatile, with a standard deviations around 2% per month. These estimates imply that the highest (annualized) Sharpe ratios are associated with the CMA (0.57) and UMD (0.53) factors.

The momentum factor shows negative skewness (-1.34), which combined with a very high kurtosis (13.32), implies relevant downside risk (see Barroso and Santa-Clara 2015 and Daniel and Moskowitz 2016). This is confirmed by the large cumulative loss (maximum drawdown) of -57% for UMD, which is on par with the cumulative losses observed for the market and SMB factors. At the other end, the investment factor registers the smallest maximum drawdown in magnitude (-17%). The profitability factor has a large kurtosis (15.26), but the negative skewness is smaller in comparison with the momentum and market factors, thus resulting in lower downside risk (-41%).

The correlations among the different monthly factors, which are tabulated in the appendix, show that in most cases, the market factor is not highly correlated with the other equity factors. The main exception arises for the asset growth factor, which shows a moderate negative correlation with the market factor (-40%).

4 Estimating the Risk-Return Trade-Off

In this section, we assess the predictive role of realized factor variances for factor risk premia.

4.1 Factor Risk-Return Trade-Off

We start by evaluating the direct risk-return trade-off associated with each equity factor. Specifically, we estimate by OLS the following (one-month ahead) univariate predictive regression:

$$F_{t+1} = \alpha + \theta RV_t + \varepsilon_{t+1}. \tag{17}$$

We estimate this regression by including and excluding the intercept. According to the theoretical two-factor model presented in Section 2, the intercept should be equal to zero if the model is perfectly specified, that is, the pricing error is zero. Imposing this restriction is likely to produce a more powerful empirical test of our forecasting model and originate more plausible estimates of the slope parameter. The statistical significance of the estimated parameters is assessed by using heteroskedasticity-robust t-ratios (White 1980). In addition to the standard double-sided p-values, we also rely on single-sided p-values to assess the significance of the slope estimates. The reason is that the sign of the slopes is restricted to be positive by the theory, as shown in Section 2.

The results displayed in Table 3 (Panel A) indicate a negative risk-return trade-off for the market factor, with marginal significance (10% level). This finding is in line with part of the related literature that shows a negative aggregate risk-return relation (e.g., Campbell 1987, Nelson 1991, Glosten, Jagannathan, and Runkle 1993, Whitelaw 1994, Brandt and Kang 2004, among others).[11] The estimated risk-return trade-off for UMD is also negative, with strong statistical significance (5% level).[12] Further, the fit of the relation is significantly larger than for the equity premium (R^2 above 6% compared to only 0.73% for RM).

The estimated slopes for SMB and HML are negative and positive, respectively. Yet, in both cases there is no statistical significance for those estimates and the R^2 values are around zero. In comparison, the results for both RMW and CMA are somewhat different. The estimated slopes

[11] On the other hand, several studies do not find a significant positive market risk-return trade-off (e.g., French, Schwert, and Stambaugh 1987, Baillie and DeGennaro 1990, Campbell and Hentschel 1992, Bollerslev and Zhou 2006, among others).

[12] The negative slope for the momentum factor is consistent with the evidence provided in Charoenrook and Conrad (2008) for a shorter sample. The negative risk-tradeoff for momentum is also consistent with the evidence that a dynamic strategy, which scales the momentum factor by the inverse of its past realized volatility, generates higher Sharpe ratios than the original momentum factor (see Barroso and Santa-Clara 2015). Barroso, Edelen, and Karehnke (2021) recently show that this predictor (realized volatility of momentum daily returns over the previous quarter) is robust controlling for dynamic factor exposures of momentum and for institutional crowding measures (see table 5 of that paper).

in the regressions corresponding to these two factors are positive and marginally significant (10% level). If we use one-sided p-values, then the slopes are significant at the 5% level for both factors. The intercept estimates are largely insignificant in the predictive regressions associated with either RMW or CMA, which means that the corresponding pricing errors are zero. The explanatory ratios (around or close to 3%) are clearly above the fit obtained for the market risk-return relation.

To gauge the economic significance of some of the predictive slopes, a one standard deviation increase in the lagged realized variance leads to an increase in the one-month ahead factor return of 41% ($0.0007 \times 5.88 \times 100$) and 34% ($0.0005 \times 6.84 \times 100$) for RMW and CMA, respectively. Such estimates reveal that the predictive power of realized variance is economically significant in the case of those three factors.

The results in Panel B of Table 3 show that, by excluding the intercept, we obtain a clearer picture of the factor risk-return trade-off, as all slope estimates turn to be more positive or less negative. In particular, the predictive slope estimates associated with RMW and CMA are strongly significant (5% level) by using the standard two-sided p-values. On the other hand, the negative estimates of θ in the regressions for the market and momentum factors turn out to be insignificant (at the 10% level by using double-sided p-values). A one standard deviation increase in the lagged realized variance leads to a rise in the one-month ahead factor return of 44% and 40% for RMW and CMA, respectively. In sum, the results from Table 3 suggest a positive in-sample risk-return trade-off for the investment and profitability factors. In the cases of the remaining equity factors, we tend to observe a zero risk-return trade-off in statistical terms.

The significant positive slope estimates in the predictive regressions associated with RMW and CMA indicate that the (covariance) prices of risk for these three factors are positive. This satisfies the sign restriction associated with the ICAPM of Merton (1973), as discussed in Maio and Santa-Clara (2012). In particular, Cooper and Maio (2019a) provide evidence that a state variable associated with RMW forecasts a rise in the aggregate market return. On the other hand, a state variable associated with CMA predicts a decline in future stock market volatility.

4.2 Controlling for the Market

Next, we estimate the following bivariate regression,

$$F_{t+1} = \alpha + \theta_1 RCMF_t + \theta_2 RV_t + \varepsilon_{t+1}, \tag{18}$$

where $RCMF$ denotes the realized covariance between the market and factor F, which is computed as follows:

$$RCMF_t = \sum_{n=0}^{20} F_{d_t - n} RM_{d_t - n}. \tag{19}$$

This predictive regression represents an empirical time-series test of the simple two-factor ICAPM presented in Section 2. This forecasting model allows to assess if there is a positive factor risk-return tradeoff, conditional on the covariance with the market factor. As in the previous subsection, we estimate the regression with and without the intercept.

The results for the baseline bivariate regressions are reported in Table 4 (Panel A). We can see that the estimates for θ_1 are positive and above one in the cases of both RMW and CMA, yet there is no statistical significance. This suggests that the realized covariance with the market factor does not help forecasting these two factors, as well as the remaining non-market factors (all with negative, but insignificant, estimates of θ_1). In comparison, the estimates of θ_2 are significantly positive in the cases of RMW and CMA, while the corresponding intercept estimates are largely insignificant. On the other hand, the slope estimate is significantly negative in the case of UMD. For these three factors, we obtain explanatory ratios above 3%, which signals relevant economic significance.

As in the analysis with univariate predictive regressions, excluding the intercept has a large impact on the results by turning the estimates of θ_2 more positive, as shown in Panel B of Table 4. These estimates are significant at the 1% level in the case of CMA and at the 5% level in the case of RMW. If anything, the slopes associated with the realized variances of these two factors are more significant than the corresponding estimates in the corresponding univariate regressions discussed above. This suggests that the covariance with the market factor clarifies the forecasting role of the realized variances associated with these factors. The economic significance of the predictive slopes associated with the profitability and investment factors is even stronger than in the univariate case: A one standard deviation increase in the lagged realized variance leads to an increase in the one-month ahead factor return of 67% ($0.0007 \times 9.52 \times 100$) and 65% ($0.0005 \times 12.95 \times 100$) per month for RMW and CMA, respectively.

Excluding the intercept also pushes the estimates of θ_1 to become more positive in most cases. Specifically, the corresponding estimates are above three for both RMW and CMA, which represent plausible estimates for the risk aversion parameter. There is marginal significance (10% level by considering single-sided p-values) for those two factors.[13]

Overall, these results suggest a good empirical consistency with the two-factor ICAPM when it comes to forecasting either the profitability and investment factors, as indicated by the estimated slopes in the forecasting regressions. It turns out that the realized covariances (with the market factor) play a secondary role, in comparison to the own factor variances, as indicated by the weaker statistical significance associated with the first group of realized moments. However, it is the case that these covariance terms clarify the role of the own variances when it comes to predicting the future monthly returns of these factors.

4.3 Sensitivity analysis

We conduct several robustness checks to the main empirical analysis discussed above. To save space and keep the focus, the results are tabulated in the online appendix. We restrict the analysis to the univariate regressions.

First, as a robustness check to the asymptotic t-ratios, we compute pseudo t-ratios obtained from a bootstrap experiment. This bootstrap simulation produces an empirical distribution for the estimated slope in the predictive regressions that might represent a better approximation for their finite sample distribution. In this simulation, the factor and the forecasting variable are simulated (10,000 times). The null hypotheses assumes no predictability of the factor return and also that

[13] These results are consistent with the cross-sectional asset pricing literature. Specifically, tests of multifactor models on the cross-section of equity portfolio returns, which include an intercept in the cross-sectional regression, often produce negative market or consumption risk price estimates (e.g., Lettau and Ludvigson 2001; Hahn and Lee 2006; Jagannathan and Wang 2007; Lewellen, Nagel, and Shanken 2010, among others). To produce more powerful cross-sectional asset pricing tests, the intercept in the cross-sectional regression is often restricted to zero, which restores positive consumption or market risk price estimates.

the predictor (RV_t) follows an AR(1) process:

$$F_{t+1} = \mu + u_{t+1}, \quad (20)$$

$$RV_{t+1} = \psi + \phi RV_t + e_{t+1}. \quad (21)$$

This bootstrap procedure accounts for the relatively high persistence of the factor realized variance documented in Section 3. It also accounts for the correlation between the residuals associated with the factor and the predictor, thus correcting for the Stambaugh (1999) bias. The pseudo t-ratio is based on a standard error that measures the variation of the slope estimates across the 10,000 pseudo samples. The full description of the bootstrap algorithm is provided in the online appendix. The results indicate that the pseudo t-ratios for the slope estimates in the regressions associated with RMW and CMA are 2.54 and 3.16, respectively. This suggests strong statistical significance, which complements the asymptotic-based inference. Moreover, we find that the intercept estimates in the regressions for RMW and CMA are largely insignificant, as indicated by the very small pseudo t-ratios (below 1.20).

Second, we investigate whether the factor risk-return trade-off varies across the business cycle. This stems from previous evidence that the aggregate equity premium is countercyclical (e.g., Fama and French 1989 and Lustig and Verdelhan 2012). There is also some evidence that factor premia is countercyclical (e.g., Cooper and Maio 2019b).

We estimate the following predictive regressions,

$$F_{t+1} = \theta_1 NBER_t RV_t + \theta_2 (1 - NBER_t) RV_t + \varepsilon_{t+1}, \quad (22)$$

where $NBER$ denotes de dummy (1 in expansions) for the NBER business cycles. Hence, θ_1 (θ_2) denotes the risk-return trade-off in expansions (recessions).

The key finding is that the risk-return trade-off is substantially stronger within economic recessions than within economic booms. In particular, in the regressions without intercept, the estimates of θ_1 have smaller magnitudes and weaker significance (10% level based on double-sided p-values) in most cases, the sole exception being HML (t-ratio of 2.07). In comparison, the estimate of θ_2 is strongly significant (1% level) in the case of CMA. In the case of RMW, the estimate of θ_2 is significant at the 5% level by using single-sided p-values. The explanatory ratios are also larger than in the corresponding benchmark regressions. This pattern is even more pronounced in the regressions that contain the intercept. Therefore, these results suggest that the positive risk return trade-off associated with both RMW and CMA, documented above, is more relevant during periods of economic slowdown.

Third, we estimate the predictive regressions for a sample that ends in 2007. The goal is to assess the impact of the 2007-09 Great Recession on our results. It is well known that such event has caused a substantially rise in the volatility of stock returns and equity factors (e.g., Barroso and Santa-Clara 2015 and Daniel and Moskowitz 2016). The results show that the slope estimate in the regression associated with CMA is strongly significant (5% level), in line with the results obtained for the full sample. In the case of RMW, we observe a weaker risk-return trade-off than in the full sample period, as the coefficient estimate of 5.89 becomes significant at the 10% level (t-ratio of 1.74, albeit there is significance at the 5% level based on single-sided p-values). When the intercept is included in the predictive regressions, the t-ratios for the slope estimates register a decline in the cases of these two factors, yet we still obtain statistical significance (5% or 10% level) by employing single-sided p-values. Further, the intercept estimates are largely insignificant. These results suggest that the risk-return trade-off associated with the profitability factor became more important in recent years.[14]

Fourth, we run the following regressions,

$$F_{t+1} = \alpha + \theta \widehat{RV}_t + \varepsilon_{t+1}, \tag{23}$$

where $\widehat{RV}_t \equiv \phi RV_t$ represents the fitted value for the AR(1) process applied to RV_{t+1}. Untabulated results show that the slope estimates associated with RMW and CMA register an increase in magnitude relative to the baseline setting.

Fifth, we estimate jointly the aggregate risk-return tradeoff and the corresponding relationship for each non-market factor. The objective is to conduct a more comprehensive test of the two-factor ICAPM, in line with Guo, Savickas, Wang, and Yang (2009). More specifically, we run the

[14] By conducting the estimation over the 1963–1997 sample period, we confirm the evidence in Guo, Savickas, Wang, and Yang (2009) that there is a significant positive risk-return tradeoff associated with HML. This finding suggests that such relationship has declined substantially in recent years.

following pair of regressions (with no intercept),

$$F_{t+1} = \theta_1 RCMF_t + \theta_2 RV_{F,t} + \varepsilon_{F,t+1}, \tag{24}$$

$$RM_{t+1} = \theta_1 RV_{M,t} + \theta_2 RCMF_t + \varepsilon_{M,t+1}, \tag{25}$$

where $RV_{F,t}$ and $RV_{M,t}$ denotes the realized variance associated with the non-market and market factor, respectively. We use Pooled OLS (with heteroskedasticity-robust t-ratios) to estimate the pair of regressions in which the slopes are fixed across equations. Untabulated results indicate positive and significant (at the 5% or 1% level) estimates of θ_2 for both RMW and CMA. However, the estimates of θ_1 are largely insignificant in all cases, which should be related with the insignificant market risk-return tradeoff documented above.

5 International Evidence

In this section, we examine the factor risk-return trade-off in international stock markets.

We restrict our attention to the factors included in the international version of the Fama–French five-factor model estimated in Fama and French (2017). We present results for both a World portfolio (Global) and a World portfolio excluding U.S. (Global ex U.S.). We also show results for four different major regions: North America, Europe, Japan, and Asia-Pacific (excluding Japan). The sample is 1990:07 to 2016:12. All the monthly and daily data on the international factors are retrieved from Kenneth French's data library.

The results for the one-period ahead predictive regressions are presented in the online appendix. To keep the focus, we discuss only the results for the regressions without intercept. Starting with the results for the Global market, the slopes associated with the value and investment factors are positive and strongly significantly (at the 5% level), with explanatory ratios above 5% in both cases. In the case of RMW, we also obtain a significant (5% level) positive slope estimate, albeit the fit is modest (R^2 of 0.52%). Critically, by excluding the U.S. market, it turns out that the positive risk-return trade-off remains significant only in the case of HML, with an explanatory ratio close to 3%. The slope associated with RMW is estimated positively in the global ex. U.S. portfolio, but there is only marginal statistical significance (10% level by considering a single-sided

p-value). On the other hand, the positive estimate of θ is largely insignificant in the case of CMA (t-ratio of 1.23). These results suggest that the evidence of a positive risk-return trade-off for the investment and profitability Fama-French factors is considerably weaker in international markets in comparison to the U.S. market. However, the lower statistical significance of the predictive slope estimates can be caused by the substantially shorter sample employed in the international tests.

Across regions, most predictive slopes are insignificant (at the 10% level), when we rely on double-sided p-values. Among the exceptions, we have the positive coefficients in the regressions for RMW in the European and North-American markets. We also observe a positive risk-return trade-off for CMA in the European market and for HML in the Asia-Pacific region. If we consider single-sided p-values, we obtain significant positive slope estimates for CMA in the North-America region, as well as for HML in both the North-America and Europe regions. The evidence of a risk-return trade-off is absent in the Japanese market, as shown by the R^2 estimates around zero and clear statistical insignificance. We find no evidence of a significantly positive market risk-return trade-off in all four regions.[15]

We also compute the bivariate regressions, which include the realized covariances with the market factor, for the international markets. The results displayed in the online appendix show that the estimates of θ_2 are not very different to the corresponding estimates of θ in the single regressions for the Global and Global ex U.S. factors. The main difference occurs for the Global factors, in which case the estimate of θ_2 associated with RMW becomes more significant (1% level), while an opposite pattern holds for CMA (with the corresponding slope estimate being now significant at the 10% level). Under both versions of the global factors, the estimates of θ_1 are largely insignificant in all cases, which shows that the covariance terms have no predictive power for factor risk premia.

In what concerns the four regions, we observe that the estimate of θ_2 for RMW in the North America region becomes more significant (5% level) in comparison to the univariate regression case. An opposite pattern exists for the slope estimates associated with RMW and CMA in the European market, which become either insignificant (RMW, albeit marginally so, with a t-ratio of 1.64) or less significant (CMA, with a t-ratio of 1.89). Hence, in those cases, adding the covariance

[15]By assuming that the risk-return trade-off changes over time, Salvador, Floros, and Arago (2014) find stronger evidence of a positive trade-off in European stock markets.

with the market factor has a downwards effect on the predictive power associated with the own factor variance. Nonetheless, the estimates of θ_1 tend to be insignificant in most cases across the four regions. The few exceptions occur in the regression associated with RMW in the North America region (estimate around 4) and in the regression associated with HML in the European region (negative estimate), in which cases there is significance at the 10% level.

Overall, the results of this section suggest that the positive risk-return trade-off associated with the profitability and investment factors is less robust among international equity markets. Still, there is some evidence that such trade-off exists in the North-American and European markets. On the other hand, the covariance with the market factor does not help predicting factor risk premia in most cases.

6 Out-of-Sample Evidence

In this section, we estimate the out-of-sample risk-return trade-off for each equity factor. In light of the results documented in Section 4, this analysis allows one to assess the parameter instability over time in the forecasting regressions by relying on recursive samples. Moreover, it also enables to mimic the eventual behavior of a forecaster in real time.[16] The downside of the out-of-sample analysis relies on the low statistical power of "out-of-sample" regressions as a result of the small sample size, especially for the first sub-samples within the evaluation period (see Inoue and Kilian 2004 and Cochrane 2008 for a discussion). Therefore, the results of this section should be interpreted with some caution.

6.1 Predictability Results

To assess the out-of-sample predictability of realized volatility, the null (or restricted) model is a regression containing only a constant in which the best forecast of the factor is the corresponding

[16] An incomplete list of papers that analyze the out-of-sample predictability of stock market volatility for market returns in the context of predictive regressions includes Guo (2006), Guo and Savickas (2006), Goyal and Welch (2008), Rapach, Strauss, and Zhou (2010), Maio (2014, 2016), and Andreou, Kagkadis, Maio, and Philip (2021).

historical average,

$$H_0 : F_{t+1} = \alpha_s + u_{t+1}, \tag{26}$$
$$H_a : F_{t+1} = \alpha_s + \theta_s RV_t + v_{t+1},$$

with $t = 1, ..., s$ and $s = T^* - 1, T^*, ..., T - 2$ and T denoting the last observation in the sample. T^* denotes the month of the last observation for F_{t+1} included in the in-sample period used for the first regression. Using the subscript s makes clear that the coefficient estimates are specific to a given window that contains information up to time $s + 1$. H_a corresponds to the alternative (or unrestricted) model, which represents the predictive regression containing the realized factor variance (RV_t) as a predictor. It is clear from these regressions that the restricted forecasting model is nested in the unrestricted model.[17]

The monthly forecasting errors for the out-of-sample period ($T^* + 1, T^* + 2, ..., T$) are defined by

$$\begin{aligned} F_{t+1} &= \widehat{\alpha}_s + \widehat{u}_{t+1}, \\ F_{t+1} &= \widehat{\alpha}_s + \widehat{\theta}_s RV_t + \widehat{v}_{t+1}, t = T^*, T^* + 1, ..., T - 1, \end{aligned} \tag{27}$$

for the restricted and unrestricted models, respectively.

To be consistent with the analysis in Section 4, we also consider the case in which the main predictive regression does not contain an intercept:

$$H_0 : F_{t+1} = \alpha_s + u_{t+1}, \tag{28}$$
$$H_a : F_{t+1} = \theta_s RV_t + v_{t+1}.$$

In this alternative setup, the null forecasting model (historical average) does not represent a restricted version of the alternative or "unrestricted" model.

The first measure to assess predictive performance is the out-of-sample coefficient of determi-

[17]This is the main reason why the forecasting regressions estimated in this section include the intercept.

nation,

$$R^2_{OS} = 1 - \frac{MSE_U}{MSE_R}, \tag{29}$$

where $MSE_U = \frac{1}{T_{OS}} \sum_{t=T^*}^{T-1} \widehat{v}_{t+1}^2$ denotes the mean-squared forecast error associated with the unrestricted model and MSE_R represents the same for the restricted model. $T_{OS} = T - T^*$ is the number of observations used in the evaluation (out-of-sample) period. The out-of-sample R^2 is positive if $MSE_U < MSE_R$, that is, the squared forecast errors associated with the unrestricted model are lower than those associated with the restricted model.

The second evaluation metric is the constrained out-of-sample coefficient of determination, denoted by R^2_{COS1}, which is proposed by Campbell and Thompson (2008). This measure is based on forecasting residuals from constrained regressions, that is, whenever the unrestricted model (predictive regression including RV) forecasts a negative factor realization for next period, such estimate is truncated to zero. In that sense, the predictive regressions rule out negative factor risk premia, consistent with theory.

We also compute a second constrained explanatory ratio (denoted by R^2_{COS2}), which is based on a restriction on the sign of the slope estimate ($\widehat{\theta}_s$) from the predictive regression. That is, whenever the regression produces a negative slope estimate, such estimate is truncated to zero. Such restriction comes in addition to the positive forecast constraint associated with R^2_{COS1}. In the benchmark results, the first recursive regression (in-sample period) uses data from 1967:02 to 1987:01 (240 months) so that the evaluation period starts in 1987:02.

The results for the out-of-sample evaluation metrics are presented in the online appendix. When the regression includes an intercept it follows that the R^2_{OS} estimates are negative across most equity factors. The sole exception is the momentum factor, with an explanatory ratio of 2.16%. Critically, when we impose the restriction of positive fitted factor risk premia, it turns out that the explanatory ratio becomes positive in the case of RMW (2.37%). Hence, the restriction of positive forecasted factor returns is clearly binding in the case of the profitability factor. Regarding the other factors, imposing the positivity constraint on the regression-based forecasts (associated with realized volatility) does not improve the forecasts, as the explanatory ratios remain negative in most cases. In the case of UMD, imposing such constraint actually hurts the predictive performance of realized variance, as the explanatory ratio declines to a value close to zero (0.25%). Imposing a constraint on the sign of the slope estimate produces a positive, albeit lower, explanatory ratio (1.14%) in the case of RMW, which shows that this restriction is relevant for the profitability factor. Such constraint also deteriorates the forecasting performance of UMD, as the corresponding R^2_{COS2} estimate becomes negative.

If the intercept is excluded from the forecasting regression, we no longer observe a positive explanatory ratio when forecasting the momentum factor ($R^2_{OS} = -5.61\%$). Imposing the positivity constraint on the regression-based forecasts of UMD deteriorates further the forecasting performance, as indicated by the more negative estimate of R^2_{COS1}. Critically, RMW turns out to be the sole factor with positive forecasting performance (1.94%), when we impose the restriction of a positive forecast.

As a robustness check, we use a longer in-sample window for the first regression (360 months), implying that the first forecast starts in 1997:02. The results are qualitatively similar to the results in the benchmark case (with a shorter in-sample period), the main difference being the positive explanatory ratio associated with the market factor (0.32%). Specifically, the estimates of both R^1_{COS2} (2.45%) and R^2_{COS2} (1.21%) associated with RMW are marginally higher than in the benchmark case, thus confirming the relevance of the positivity restrictions for the profitability factor. Yet, such constraints do not help the predictive performance of most of the other factors, as indicated by the negative R^1_{COS2} and R^2_{COS2} estimates observed in most cases. By excluding the intercept, we obtain a marginally better forecasting performance in the case of RMW, with a R^2_{COS1} estimate of 2.68%, while the explanatory ratios associated with the market factor become negative. On the other hand, we obtain a positive explanatory ratio in the case of CMA, albeit the magnitude is relatively small (0.19%).

In sum, these results indicate that realized factor variance helps to forecast (out-of-sample) the profitability factor, once the positivity constraints are imposed. There is also some evidence of predictability for the market, momentum, and investment factors, as indicated by positive explanatory ratios, although these estimates are either more modest or less robust. These findings are partially in line with existing evidence showing that it is substantially more difficult to forecast stock market returns out-of-sample than in-sample (e.g., Goyal and Welch 2008).

6.2 Economic Significance: Benchmark Strategy

We evaluate the economic significance of the out-of-sample estimated risk-return trade-off for each non-market factor. More concretely, we construct binary trading strategies based on the out-of-sample factor return predictability, in line with the work of Breen, Glosten, and Jagannathan (1989), Goyal and Santa-Clara (2003), Maio (2014, 2016), among others.

Specifically, at each time $t = T^*, T^* + 1, ..., T - 1$, we compute the forecasted factor return for next period as

$$\widehat{F}_{t+1} = \widehat{\alpha}_s + \widehat{\theta}_s RV_t, \tag{30}$$

where $s = t - 1$ denotes the month for the last observation of RV employed in the regression that originates the parameter estimates. We consider both the case of a regression containing the intercept and a regression excluding the intercept ($\widehat{\alpha}_s = 0$).

The trading strategy invests 100% in the stock index plus a dynamic exposure to the (non-market) equity factor. The strategy goes long the factor (with a weight of 150%) if the forecasted return is positive, otherwise it shorts the factor (with a weight of -150%). In symbols, the trading strategy can be represented as

$$\omega_t = \begin{cases} 1.5 \; if \; \widehat{F}_{t+1} \geq 0 \\ -1.5 \; if \; \widehat{F}_{t+1} < 0 \end{cases}, \tag{31}$$

where ω_t denotes the exposure to the factor.

At time $t + 1$, the realized return for the trading strategy is given by

$$R_{p,t+1} = \omega_t F_{t+1} + R_{m,t+1}, \tag{32}$$

where $R_{m,t+1}$ denotes the market return and F_{t+1} is the factor realization at $t + 1$.[18] By iterating this process forward, we create a time-series of realized returns for the active strategy.

The corresponding passive strategy is a passive "buy-hold" strategy that simply allocates 100% to the stock index:

$$\widetilde{R}_{p,t+1} = R_{m,t+1}. \tag{33}$$

The dynamic strategy presented above is suitable for an investor who is skeptical about investing

[18] In this section, R_m denotes the return (rather than the excess return) on the stock index.

in the factor strategies, that is, such investor does not believe that these factors provide a positive unconditional premium. Thus, in average he/she holds the market portfolio, but is willing to time the factors (in both directions) according to the signals of the predictive regressions.

Following Campbell and Thompson (2008), Ferreira and Santa-Clara (2011), Maio (2013, 2014), among others, to evaluate the economic significance of the out-of-sample risk-return trade-off, we compute the change in average utility:

$$\Delta U = \mathrm{E}\left(R_{p,t+1}\right) - \mathrm{E}\left(\widetilde{R}_{p,t+1}\right) + \frac{\gamma}{2}\left[\mathrm{var}\left(\widetilde{R}_{p,t+1}\right) - \mathrm{var}\left(R_{p,t+1}\right)\right]. \tag{34}$$

This metric assumes a simple mean-variance utility function,

$$U\left(R_{p,t+1}\right) = \mathrm{E}\left(R_{p,t+1}\right) - \frac{\gamma}{2}\,\mathrm{var}\left(R_{p,t+1}\right), \tag{35}$$

where γ represents the level of relative risk aversion. ΔU can be interpreted as the annual "fee" that an investor is willing to pay in order to invest into a trading strategy (instead of holding the corresponding passive strategy). To assess the sensitivity of the results to γ, we calibrate three different values of risk aversion (three, five, and ten).

We also compute standard performance evaluation measures for the trading strategy: mean return, standard deviation of the return, annualized pseudo Sharpe ratio (mean return divided by the volatility), skewness, kurtosis, and maximum drawdown. This last metric represents the maximum cumulative loss observed during the lifetime of the strategy. Further, we report the fraction of months in which the dynamic strategy takes a long position in the equity factor.

The results associated with the trading strategy are presented in Table 5 (Panel A). The passive strategy produces an average return of 0.87% per month, which combined with a volatility of 4.39% per month, yields an annual Sharpe ratio of 0.69. With the exception of SMB, all factor trading strategies produce higher Sharpe ratios than the passive strategy. This arises from the higher mean returns that more than compensate for the higher volatilities (in most cases) relative to the buy-hold rule. In particular, the active strategies corresponding to both RMW and CMA generate annual Sharpe ratio above one. Among these two factors, the strategy associated with CMA clearly dominates the passive strategy as it produces both a higher mean return (1.31%) and a marginally lower volatility (4.31%).

The change in certainty equivalent estimates confirm that the strategies associated with the investment and profitability factors have the best overall performance: The annualized fees are above 5% in all cases, which indicates large economic significance. While in the case of RMW these estimates decrease slightly with the level of risk aversion (from 8.09% for $\gamma = 3$ to 6.65% for $\gamma = 10$), we do not observe the same pattern for CMA (from 5.34% for $\gamma = 3$ to 5.64% for $\gamma = 10$).

Among the remaining factors, we obtain positive ΔU estimates in the case of HML, although the gain in average utility becomes negative for high levels of risk aversion ($\gamma = 10$). We also verify that the dynamic strategy associated with the momentum factor yields negative ΔU estimates for moderate and high levels of risk aversion (-18% for $\gamma = 10$). This stems from the relatively large volatility of the active momentum strategy (8.31% per month), which becomes severely penalized at high levels of risk aversion.

Regarding the other evaluation metrics, it turns out that most dynamic strategies have negative skewness, the exceptions being the strategies associated with UMD and RMW. The strategies associated with SMB, HML, RMW, and especially momentum exhibit large kurtosis (around or above 6). The combination of negative skewness with large kurtosis indicates significant downside risk, which is especially relevant in the case of the dynamic strategy corresponding to the value factor. Indeed, the estimates for MDD are partially consistent with those estimates for the third and fourth empirical moments: The maximum cumulative loss is obtained for HML (around -70%), followed by UMD (-64%). At the other end of the spectrum, the strategy associated with RMW (-30%) shows the lowest cumulated loss. Hence, the trading strategies corresponding to the investment and (especially) profitability factors tend to have have lower downside risk than the strategies associated with the value-growth and momentum factors. However, the former group of factor dynamic strategies differs considerably in the factor exposures over time: while in the case of CMA the dynamic rule goes long the factor at all months, it turns out that in the case of RMW the dynamic strategy goes long only about 91% of the time (the lowest percentage among all five factors, along with UMD). Results displayed in the online appendix show that the negative factor weights in the case of RMW occur mainly in the first half of the sample, in contrast to the pattern observed for UMD.

Panel B of Table 5 contains the results of the factor trading strategies when the predictive regression does not include the intercept. Excluding the intercept has no impact on the strategy associated with CMA. On the other hand, there is a decline in the performance of the dynamic strategy associated with RMW (in comparison to the benchmark case), as indicated by Sharpe ratio of 0.91 (versus 1.16) and ΔU estimates varying from 3.86% (compared to 8.09%) at $\gamma = 3$ to 2.78% (compared to 6.65%) at $\gamma = 10$. Nonetheless, these estimates still indicate large economic significance.

We conduct several robustness checks to the results associated with the benchmark strategy. To save space and keep the focus, these results are tabulated in the online appendix and we consider only the baseline setup in which the predictive regression includes the intercept. First, we use an in-sample period of 30 years so that the first forecast occurs for 1997:02. The results show that the utility gains are around or above 8% (5%) in the case of RMW (CMA).

Second, we define an alternative version of the trading strategy,

$$\omega_t = \begin{cases} 2\ if\ \widehat{F}_{t+1} \geq 0 \\ -2\ if\ \widehat{F}_{t+1} < 0 \end{cases}, \tag{36}$$

which imposes a more levered position on each factor (2 versus 1.5). The results are qualitatively similar to the benchmark results. Specifically, the Sharpe ratios associated with the trading strategies for the investment and profitability factors are above one in both cases. Further, the utility gains for the strategies corresponding to these factors are around or above 5% in most cases, the exception being the case of RMW when $\gamma = 10$ (ΔU of 4.82%).[19]

Third, we conduct an alternative version of the trading strategy in which the slope estimates (in the forecasting regressions) are truncated to zero whenever they assume negative values. The results are quite similar to those in the benchmark case. In the case of CMA, we obtain the same estimates as in the baseline dynamic strategy. In the case of RMW, we obtain slightly lower estimates of ΔU (between 4.78% and 5.73%).

Fourth, we compute a new version of the dynamic strategy by employing bivariate forecasting regressions, which include the realized covariance of each factor with the market factor. The results are qualitatively similar to those in the benchmark strategy. In particular, we obtain slightly lower

[19]Untabulated results show that using an unlevered position in the equity factor ($\omega_t = 1, -1$) produces significant utility gains (above 4%) in the cases of the profitability and investment factors.

ΔU estimates when the equity factors are RMW and CMA. We also observe an improvement in the profitability of the trading strategy associated with HML in comparison to the benchmark case.[20]

6.3 Economic Significance: Alternative Strategy

Next, we define an alternative dynamic strategy that is similar to the benchmark strategy presented above, except that the investor does not short the factor if the forecasted return is negative:

$$\omega_t = \begin{cases} 1.5 \; if \; \widehat{F}_{t+1} \geq 0 \\ 0 \; if \; \widehat{F}_{t+1} < 0 \end{cases} . \tag{37}$$

The corresponding passive strategy invests 100% in the stock index and takes a permanent positive weight (long position) in the factor:

$$\widetilde{R}_{p,t+1} = R_{m,t+1} + 1.5F_{t+1}. \tag{38}$$

We note that, in contrast to the benchmark strategy, each factor has a different passive strategy. This strategy may be suitable for an investor who has a positive prior on each of the factors (positive factor premium), that is, he/she wants to maintain a positive average factor exposure. Hence, the investor wants to evaluate the benefit of timing the positive factor exposure against a "buy-hold" factor exposure.

The results for the alternative dynamic strategy are presented in the online appendix. The active strategies associated with RMW (1.24) and UMD (0.94) generate higher Sharpe ratios than the corresponding buy-hold factor strategies. Consequently, the utility gains associated with these two dynamic strategies are positive and economically significant at all levels of risk aversion: the estimates vary from 2.03% to 3.75% in the case of RMW and from 3.95% to 10.19% in the case of UMD. The trading strategies corresponding with most of the remaining factors, including CMA, coincide with the corresponding passive rules. The reason is that in these cases, the corresponding active strategy goes long in the factor in every period.

[20]Untabulated results show that the trading strategy associated with the bivariate regressions with no intercept generates positive gains in utility for RMW and CMA.

When the intercept is removed from the predictive regressions containing realized variance, we observe a decline in the profitability of the trading strategies associated with both UMD and RMW, as indicated by the lower certainty equivalent estimates. This pattern is especially evident in the case of the momentum factor. Nonetheless, there are still economically significant gains, especially the case of RMW, as suggested by the ΔU estimates of 0.76% and 2.43% for $\gamma = 5$ and $\gamma = 10$, respectively. Overall, the results for the alternative trading strategy indicate that an investor who takes a permanent long position in the factors can benefit from timing this exposition (based on the forecasting power of realized factor variances) in the cases of the profitability and momentum factors.

Taken together, the results of this section show that the out-of-sample forecasting power (of realized volatility for future returns), despite being relatively modest, tends to be robust and economically significant in the cases of the profitability and investment factors. In comparison, such economic significance does not exist (or is substantially less robust) for the remaining equity factors.

7 Conclusion

We contribute to the risk-return trade-off literature by examining the risk-return relation among equity factors. In the same vein that the market risk-return trade-off is consistent with the conditional CAPM, the factor risk-return relation is consistent with conditional two-factor models that can be interpreted as empirical applications of the ICAPM of Merton (1973). Specifically, we employ conditional two-factor models containing the market and each of the non-market factors employed in the multifactor models of Carhart (1997) and Fama and French (2015, 2016) to motivate and explain our empirical tests. According to those models, there should exist a positive relation between the risk premium and conditional variance of the size, value, momentum, profitability, and investment risk factors, controlling by the conditional covariance with the market factor. To proxy for the unobserved conditional factor variances and covariances, we compute monthly realized factor variances and covariances based on daily factor observations, an approach that is widely adopted in the risk-trade-off literature.

By using monthly data from 1967 to 2016, the results of univariate predictive regressions suggest a positive in-sample risk-return trade-off for the profitability (RMW) and investment (CMA) factors of Fama and French (2015). This positive risk-return trade-off is economically significant as indicated by the magnitudes of the slopes associated with lagged factor variance. On the other hand, we do not find a significant positive risk-return trade-off for the remaining equity factors, including the market and momentum factors.

By estimating bivariate predictive regressions, which include the realized covariance with the market factor, we obtain a good empirical consistency with the two-factor ICAPM when it comes to forecasting either CMA or RMW: In both cases, we obtain significant positive slope estimates associated with the realized factor variances, in addition to obtaining plausible estimates of the risk aversion parameter. It turns out that the realized covariances (with the market factor) play a secondary role, in comparison to the own factor variances, as indicated by the weaker statistical significance (associated with the first group of realized moments). However, it is the case that these covariance terms clarify the role of the own variances when it comes to predicting the future monthly returns of these factors. Overall, our results indicate that a positive risk-return trade-off is much more pervasive for specific segments of the U.S. stock market (e.g., profitability and investment sorted portfolios) than for the market as a whole.

To put these results in perspective, we examine the factor risk-return trade-off in international stock markets over a shorter period (1990 to 2016). The results suggest that the positive risk-return trade-off associated with the profitability and investment factors is less robust among international equity markets. Still, there is some evidence that such trade-off exists in the North-American and European markets. On the other hand, the covariance with the market factor does not help predicting factor risk premia in most cases.

In the last part of the paper, we examine the economic significance of the out-of-sample forecasts associated with factor realized variances for future factor returns. To achieve that goal, we construct a trading strategy that relies on such predictability. The results indicate that the out-of-sample forecasting power (of realized volatility for future factor returns) is economically significant in the cases of the profitability and investment factors. Specifically, the annual pseudo Sharpe ratios are above one in both cases compared to 0.69 for the passive strategy that only invests in the stock market index. Moreover, the utility gains associated with the dynamic strategies corresponding to those two factors are above 3.5% per year in most cases. In comparison, such economic significance does not exist or is not robust for the other equity factors. By using an alternative trading strategy, which explores only positive factor risk premia, we obtain economically significant gains in the case of the profitability factor.

This paper exclusively looks at the conditional factor risk-return trade-off over time. A natural extension of our work is to incorporate the restrictions that emanate from our time-series tests into cross-sectional asset pricing tests of linear factor models (containing the very same traded factors). This is beyond the scope of our paper and is left for future work.

Table 1: Descriptive Statistics for Realized Variances

This table reports descriptive statistics for the realized variance of each factor. RM, SMB, HML, UMD, RMW, and CMA denote the Fama–French–Carhart market, size, value-growth, momentum, profitability, and investment factors, respectively. The sample is 1967:02–2016:12.

	Mean(%)	SD(%)	Min.(%)	Max.(%)
RM	0.22	0.41	0.02	5.48
SMB	0.06	0.11	0.01	2.14
HML	0.06	0.11	0.00	1.03
UMD	0.41	1.10	0.02	13.73
RMW	0.03	0.07	0.00	0.88
CMA	0.03	0.05	0.00	0.58

Table 2: Persistence of Realized Variances

This table reports the results for AR(1) process for the realized variance of each factor. RM, SMB, HML, UMD, RMW, and CMA denote the Fama–French–Carhart market, size, value-growth, momentum, profitability, and investment factors, respectively. ϕ denotes the predictive slope, while t represents the corresponding GMM-based t-ratio (in parentheses). R^2 denotes the coefficient of determination. t-ratios marked with *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels. The sample is 1967:02–2016:12.

	ϕ	t	R^2(%)
RM	0.53	(3.45***)	28.20
SMB	0.35	(2.11**)	11.91
HML	0.73	(8.52***)	53.98
UMD	0.77	(5.71***)	59.77
RMW	0.77	(4.54***)	59.42
CMA	0.71	(5.29***)	50.57

Table 3: Risk-Return Trade-Off: Univariate Regressions

This table reports the results for regressions of each factor on its lagged realized variance. *RM*, *SMB*, *HML*, *UMD*, *RMW*, and *CMA* denote the Fama–French–Carhart market, size, value-growth, momentum, profitability, and investment factors, respectively. α (θ) denotes the intercept (slope) estimate, while t represents the corresponding GMM-based t-ratio (in parentheses). R^2 is the coefficient of determination. t-ratios marked with *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels. In Panel B, the regressions do not include the intercept. The sample is 1967:02–2016:12.

	α	t	θ	t	$R^2(\%)$
Panel A: intercept					
RM	0.007	(3.67***)	−0.93	(−1.66*)	0.73
SMB	0.002	(1.58)	−0.41	(−0.24)	0.02
HML	0.003	(2.73***)	0.67	(0.40)	0.07
UMD	0.011	(5.25***)	−1.02	(−2.25**)	6.80
RMW	0.001	(0.75)	5.88	(1.70*)	3.04
CMA	0.001	(1.11)	6.84	(1.68*)	2.72
Panel B: no intercept					
RM			−0.21	(−0.35)	0.29
SMB			0.40	(0.28)	−0.07
HML			1.89	(1.20)	−0.15
UMD			−0.70	(−1.51)	6.14
RMW			6.29	(1.99**)	3.03
CMA			8.05	(2.47**)	2.64

Table 4: Risk-Return Trade-Off: Bivariate Regressions

This table reports the results for regressions of each factor on its lagged realized variance (captured by θ_2) and the lagged realized covariance with the market factor (θ_1). *SMB*, *HML*, *UMD*, *RMW*, and *CMA* denote the Fama–French–Carhart size, value-growth, momentum, profitability, and investment factors, respectively. α denotes the intercept estimate. t represent the GMM-based t-ratios (in parentheses). R^2 is the coefficient of determination. t-ratios marked with *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels. In Panel B, the regressions do not include the intercept. The sample is 1967:02–2016:12.

	α	t	θ_1	t	θ_2	t	$R^2(\%)$
Panel A: intercept							
SMB	0.002	(1.61)	−0.59	(−0.33)	−0.63	(−0.33)	0.05
HML	0.002	(1.68*)	−2.06	(−1.29)	1.47	(0.76)	0.89
UMD	0.011	(5.70***)	−0.23	(−0.20)	−1.08	(−2.44**)	6.82
RMW	0.000	(0.36)	3.58	(1.23)	9.18	(2.11**)	3.53
CMA	0.001	(0.88)	3.70	(1.24)	11.45	(1.94*)	3.12
Panel B: no intercept							
SMB			−0.33	(−0.18)	0.29	(0.17)	−0.05
HML			−2.55	(−1.72*)	2.45	(1.49)	0.75
UMD			0.73	(0.61)	−0.51	(−1.18)	5.89
RMW			3.73	(1.33)	9.52	(2.50**)	3.53
CMA			4.13	(1.47)	12.95	(2.77***)	3.06

Table 5: Benchmark Trading Strategy

This table reports the performance evaluation measures associated with the benchmark trading strategy that is based on the out-of-sample forecasting power of the realized variance for the return of each factor. SMB, HML, UMD, RMW, and CMA denote the active strategies for the Fama–French–Carhart size, value-growth, momentum, profitability, and investment factors, respectively. P denotes the passive strategy. The statistics are the average return (Mean), standard deviation (SD), annualized raw Sharpe ratio (Sharpe), Skewness (Skew.), Kurtosis (Kurt.), and maximum drawdown (MDD). "Long" denotes the fraction of months in which the active strategy goes long in the factor. $\Delta U, \gamma = 3$, $\Delta U, \gamma = 5$, and $\Delta U, \gamma = 10$ represent the annualized change in average utility associated with a risk aversion coefficient of three, five, and ten, respectively. The total sample is 1967:02–2016:12 and the out-of-sample evaluation period starts in 1987:02. In Panel B, the predictive regressions that generate the forecasts do not include the intercept.

	Mean(%)	SD(%)	Sharpe	Skew.	Kurt.	MDD(%)	Long(%)	$\Delta U(\%), \gamma = 3$	$\Delta U(\%), \gamma = 5$	$\Delta U(\%), \gamma = 10$
Panel A: intercept										
P	0.87	4.39	0.69							
SMB	1.03	7.16	0.50	−0.29	6.31	−52.74	0.99	−3.93	−7.77	−17.35
HML	1.25	5.57	0.78	−0.72	5.85	−69.75	1.00	2.36	0.96	−2.56
UMD	1.86	8.31	0.78	0.74	13.12	−63.99	0.92	2.90	−3.07	−17.99
RMW	1.60	4.77	1.16	0.30	8.60	−29.80	0.91	8.09	7.68	6.65
CMA	1.31	4.31	1.05	−0.60	4.90	−51.32	1.00	5.34	5.42	5.64
Panel B: no intercept										
P	0.87	4.39	0.69							
SMB	0.92	7.13	0.45	−0.42	6.11	−68.36	1.00	−5.18	−8.96	−18.41
HML	1.25	5.57	0.78	−0.72	5.85	−69.75	1.00	2.36	0.96	−2.56
UMD	1.52	7.97	0.66	−0.71	8.14	−66.25	0.74	−0.24	−5.55	−18.81
RMW	1.24	4.68	0.91	0.03	10.58	−29.80	0.65	3.86	3.55	2.78
CMA	1.31	4.31	1.05	−0.60	4.90	−51.32	1.00	5.34	5.42	5.64

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A Internet Appendix: Bootstrap Algorithm for Predictive Regressions

The bootstrap algorithm associated with the predictive regressions consists of the following steps:

1. The forecasting regression is estimated by OLS, and one saves the slope estimate, $\widehat{\theta}$:

$$F_{t+1} = \alpha + \theta RV_t + \varepsilon_{t+1}. \tag{A.1}$$

We also estimate by OLS the following system that imposes the joint null of no predictability of factor returns and a persistent predictor that follows an AR(1) process:

$$F_{t+1} = \mu + u_{t+1}, \tag{A.2}$$

$$RV_{t+1} = \psi + \phi RV_t + e_{t+1}. \tag{A.3}$$

The time-series of OLS residuals, $\widehat{u}_{t+1}$ and $\widehat{e}_{t+1}$, and the OLS estimates, $\widehat{\mu}, \widehat{\psi}, \widehat{\phi}$ are saved.

2. In each replication $m = 1, ..., 10000$, we construct pseudo-samples for the innovations in the factor and the predictor by drawing with replacement from the two residuals:

$$\{\widehat{u}_{t+1}^m\}, t = s_1^m, s_2^m, ..., s_T^m, \tag{A.4}$$

$$\{\widehat{e}_{t+1}^m\}, t = s_1^m, s_2^m, ..., s_T^m, \tag{A.5}$$

where the time indices $s_1^m, s_2^m, ..., s_T^m$ are created randomly from the original time sequence $1, ..., T$. Notice that the innovations in both the factor and predictor have the same time sequence to account for their contemporaneous cross-correlation.

3. For each replication $m = 1, ..., 10000$, we construct a pseudo-sample of the factor and predictor, by imposing the null:

$$F_{t+1}^m = \widehat{\mu} + \widehat{u}_{t+1}^m, \tag{A.6}$$

$$RV_{t+1}^m = \widehat{\psi} + \widehat{\phi} RV_t^m + \widehat{e}_{t+1}^m. \tag{A.7}$$

4. In each replication, we estimate the predictive regression, but using the artificial data rather than the original data:

$$F^m_{t+1} = \alpha^m + \theta^m RV^m_t + \varepsilon^m_{t+1}. \tag{A.8}$$

The initial value for RV^m_t (RV^m_0), is picked at random from one of the observations of RV_t. As a result, there is an empirical distribution of the regression slope estimates, $\left\{\widehat{\theta}^m\right\}_{m=1}^{10000}$ (as opposed to the asymptotic theoretical distribution).

5. Given the collection of 10000 slope estimates, we construct an empirical standard error,

$$se(\widehat{\theta}^m) = \sqrt{\frac{1}{10000}\sum_{m=1}^{10000}[\widehat{\theta}^m - \mathrm{E}^m(\widehat{\theta}^m)]}, \tag{A.9}$$

where $\mathrm{E}^m(\widehat{\theta}^m) \equiv \frac{1}{10000}\sum_{m=1}^{10000}\widehat{\theta}^m$ denotes the average slope across all pseudo samples. The pseudo t-ratio associated with the predictive slope is then calculated as:

$$t^*(\widehat{\theta}) = \frac{\widehat{\theta}}{se(\widehat{\theta}^m)}. \tag{A.10}$$

B Internet Appendix: Additional Results

This appendix contains results that are discussed, but not tabulated, in the main paper. The results in Tables A.1 and A.2 are discussed in Section 3. Tables A.3 to A.5 are discussed in Section 4.3. The results in Tables A.6 and A.7 are associated with Section 5. Table A.8 refers to Section 6.1. The results in Tables A.9 to A.12 are related to Section 6.2. Table A.13 refers to Section 6.3. Figures A.1 and A.2 are associated with Section 3, whereas Figure A.3 is discussed in Section 6.2.

Table A.1: Descriptive Statistics for Equity Factors

This table reports descriptive statistics for the equity factors. RM, SMB, HML, UMD, RMW, and CMA denote the Fama–French–Carhart market, size, value-growth, momentum, profitability, and investment factors, respectively. The statistics are the average return (Mean), standard deviation (SD), minimum return (Min.), maximum return (Max.), annualized raw Sharpe ratio (Sharpe), Skewness (Skew.), Kurtosis (Kurt.), and maximum drawdown (MDD). The sample is 1967:02–2016:12.

	Mean(%)	SD(%)	Sharpe	Min.(%)	Max.(%)	Skew.	Kurt.	MDD(%)
RM	0.51	4.52	0.39	−23.24	16.10	−0.52	4.81	−55.71
SMB	0.20	3.10	0.22	−16.88	21.71	0.52	8.60	−55.09
HML	0.37	2.88	0.44	−11.10	12.90	0.07	4.97	−40.89
UMD	0.66	4.31	0.53	−34.39	18.36	−1.34	13.32	−57.42
RMW	0.26	2.27	0.40	−18.72	13.51	−0.32	15.26	−41.29
CMA	0.33	2.02	0.57	−6.87	9.58	0.33	4.51	−17.28

Table A.2: Correlations of Equity Factors

This table reports the pairwise correlations among the equity factors. RM, SMB, HML, UMD, RMW, and CMA denote the Fama—French–Carhart market, size, value-growth, momentum, profitability, and investment factors, respectively. The sample is 1967:02–2016:12.

	RM	SMB	HML	UMD	RMW	CMA
RM	1.00	0.29	−0.27	−0.14	−0.24	−0.40
SMB		1.00	−0.21	−0.02	−0.42	−0.16
HML			1.00	−0.19	0.09	0.70
UMD				1.00	0.11	−0.01
RMW					1.00	−0.01
CMA						1.00

Table A.3: Risk-Return Trade-Off: Alternative Statistical Inference

This table reports the results for regressions of each factor on its lagged realized variance. RM, SMB, HML, UMD, RMW, and CMA denote the Fama–French–Carhart market, size, value-growth, momentum, profitability, and investment factors, respectively. α (θ) denotes the intercept (slope) estimate. t denotes pseudo t-ratios (in parentheses), which are obtained from a bootstrap simulation. R^2 is the coefficient of determination. In Panel B, the regressions do not include the intercept. The sample is 1967:02–2016:12.

	α	t	θ	t	R^2(%)
Panel A: intercept					
RM	0.007	(3.06)	−0.93	(−1.21)	0.73
SMB	0.002	(1.30)	−0.41	(−0.14)	0.02
HML	0.003	(2.69)	0.67	(0.50)	0.07
UMD	0.011	(5.65)	−1.02	(−3.94)	6.80
RMW	0.001	(0.67)	5.88	(2.11)	3.04
CMA	0.001	(1.14)	6.84	(2.23)	2.72
Panel B: no intercept					
RM			−0.21	(−0.32)	0.29
SMB			0.40	(0.18)	−0.07
HML			1.89	(1.47)	−0.15
UMD			−0.70	(−3.08)	6.14
RMW			6.29	(2.54)	3.03
CMA			8.05	(3.16)	2.64

Table A.4: Risk-Return Trade-Off and the Business Cycle

This table reports the results for regressions of each factor on its lagged realized variance across expansions (captured by θ_1) and recessions (θ_2). RM, SMB, HML, UMD, RMW, and CMA denote the Fama–French–Carhart market, size, value-growth, momentum, profitability, and investment factors, respectively. α denotes the intercept estimate. t represent the GMM-based t-ratios (in parentheses). R^2 is the coefficient of determination. t-ratios marked with *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels. In Panel B, the regressions do not include the intercept. The sample is 1967:02–2016:12.

	α	t	θ_1	t	θ_2	t	R^2(%)
Panel A: intercept							
RM	0.007	(3.36***)	−0.64	(−0.85)	−1.15	(−1.39)	0.79
SMB	0.002	(1.51)	−0.78	(−0.38)	1.38	(0.52)	0.15
HML	0.002	(1.21)	5.28	(1.30)	−0.58	(−0.36)	1.23
UMD	0.011	(5.55***)	−0.99	(−1.31)	−1.02	(−2.10**)	6.80
RMW	0.001	(0.67)	5.67	(1.56)	8.76	(1.75*)	3.12
CMA	0.001	(1.10)	6.20	(1.28)	9.48	(2.57**)	2.86
Panel B: no intercept							
RM			0.34	(0.35)	−0.74	(−0.87)	0.35
SMB			−0.01	(−0.01)	2.31	(0.83)	0.06
HML			6.75	(2.07**)	−0.22	(−0.14)	1.14
UMD			0.17	(0.24)	−0.85	(−1.71*)	5.58
RMW			5.99	(1.75*)	9.49	(1.95*)	3.11
CMA			7.33	(1.82*)	10.82	(2.97***)	2.78

Table A.5: Risk-Return Trade-Off: Alternative Sample

This table reports the results for regressions of each factor on its lagged realized variance. RM, SMB, HML, UMD, RMW, and CMA denote the Fama–French–Carhart market, size, value-growth, momentum, profitability, and investment factors, respectively. α (θ) denotes the intercept (slope) estimate, while t represents the corresponding GMM-based t-ratio (in parentheses). R^2 is the coefficient of determination. t-ratios marked with *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels. In Panel B, the regressions do not include the intercept. The sample is 1967:02–2007:12.

	α	t	θ	t	R^2(%)
Panel A: intercept					
RM	0.006	(2.45**)	−0.52	(−0.62)	0.12
SMB	0.002	(1.36)	−0.27	(−0.14)	0.01
HML	0.002	(1.07)	6.03	(1.56)	1.80
UMD	0.009	(4.94***)	−0.38	(−0.50)	0.17
RMW	0.001	(0.80)	5.49	(1.52)	2.82
CMA	0.002	(1.16)	6.96	(1.67*)	3.00
Panel B: no intercept					
RM			0.32	(0.31)	−0.19
SMB			0.43	(0.26)	−0.06
HML			7.32	(2.35**)	1.72
UMD			0.53	(0.73)	−0.80
RMW			5.89	(1.74*)	2.80
CMA			8.22	(2.42**)	2.91

Table A.6: Risk-Return Trade-Off: International Evidence

This table reports the results for univariate regressions (without intercept) of each factor on its lagged realized variance in international stock markets. RM, SMB, HML, RMW, and CMA denote the Fama–French market, size, value-growth, profitability, and investment factors, respectively. θ denotes the predictive slope, while t represents the corresponding GMM-based t-ratio (in parentheses). R^2 is the coefficient of determination. t-ratios marked with *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels. The sample is 1990:07–2016:12

	θ	t	R^2(%)		θ	t	R^2(%)
Panel A: Global				**Panel B: Global ex U.S.**			
RM	−0.19	(−0.16)	0.17	RM	0.06	(0.06)	−0.03
SMB	−0.06	(−0.03)	0.01	SMB	−0.29	(−0.22)	0.12
HML	11.90	(2.40**)	5.21	HML	12.34	(2.39**)	2.92
RMW	14.67	(2.53**)	0.52	RMW	8.82	(1.48)	−1.90
CMA	14.82	(2.39**)	7.03	CMA	8.39	(1.23)	1.01
Panel C: North America				**Panel D: Europe**			
RM	−0.31	(−0.38)	0.70	RM	0.09	(0.10)	−0.08
SMB	−0.40	(−0.14)	0.13	SMB	−1.33	(−1.19)	1.15
HML	4.28	(1.41)	3.57	HML	5.87	(1.61)	1.24
RMW	6.89	(1.72*)	3.95	RMW	6.32	(1.68*)	−1.77
CMA	5.73	(1.41)	4.88	CMA	10.43	(2.18**)	4.19
Panel E: Japan				**Panel F: Asia-Pacific**			
RM	0.46	(0.84)	0.32	RM	0.09	(0.10)	−0.07
SMB	1.67	(1.12)	0.47	SMB	−0.65	(−0.58)	0.03
HML	2.57	(0.72)	−0.24	HML	6.98	(2.61***)	0.49
RMW	1.69	(0.70)	0.09	RMW	1.74	(0.67)	−0.15
CMA	−0.48	(−0.13)	0.09	CMA	3.00	(1.19)	−0.40

Table A.7: Risk-Return Trade-Off with Bivariate Regressions: International Evidence

This table reports the results for bivariate regressions (without intercept) of each factor on its lagged realized variance and its lagged realized covariance with the market factor, in international stock markets. SMB, HML, RMW, and CMA denote the Fama–French size, value-growth, profitability, and investment factors, respectively. θ_1 (θ_2) denotes the predictive slope associated with the lagged covariance (variance), while t represent the corresponding GMM-based t-ratios (in parentheses). R^2 is the coefficient of determination. t-ratios marked with *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels. The sample is 1990:07–2016:12

	θ_1	t	θ_2	t	R^2(%)		θ_1	t	θ_2	t	R^2(%)
	Panel A: Global						**Panel B: Global ex U.S.**				
SMB	0.87	(0.28)	1.21	(0.22)	−0.02	SMB	1.09	(0.45)	1.20	(0.31)	0.10
HML	−1.98	(−0.56)	10.79	(2.40**)	5.39	HML	−2.32	(−0.57)	11.22	(2.35**)	3.21
RMW	2.97	(0.69)	18.01	(3.01***)	0.75	RMW	−3.19	(−0.76)	7.27	(1.10)	−1.77
CMA	2.64	(0.66)	18.29	(1.90*)	7.31	CMA	−2.52	(−0.87)	4.48	(0.48)	1.52
	Panel C: North America						**Panel D: Europe**				
SMB	1.58	(0.67)	0.09	(0.03)	0.23	SMB	0.75	(0.42)	−0.34	(−0.11)	1.18
HML	−1.01	(−0.52)	3.79	(1.44)	3.69	HML	−4.61	(−1.83*)	5.95	(1.66*)	4.27
RMW	4.19	(1.66*)	10.39	(2.26**)	4.77	RMW	−2.60	(−0.78)	5.58	(1.64)	−1.42
CMA	5.32	(1.08)	10.30	(1.39)	5.68	CMA	0.88	(0.33)	11.69	(1.89*)	4.26
	Panel E: Japan						**Panel F: Asia-Pacific**				
SMB	0.06	(0.04)	1.73	(0.74)	0.47	SMB	0.75	(0.52)	0.26	(0.11)	0.09
HML	0.07	(0.02)	2.60	(0.64)	−0.24	HML	1.99	(0.79)	7.95	(2.17**)	0.97
RMW	2.92	(1.28)	2.75	(0.99)	1.15	RMW	0.26	(0.07)	1.92	(0.67)	−0.16
CMA	0.22	(0.11)	−0.36	(−0.09)	0.08	CMA	−1.21	(−0.50)	1.37	(0.37)	−0.04

Table A.8: Out-of-Sample Predictability

This table presents out-of-sample evaluation statistics for the risk-return trade-off associated with each factor. RM, SMB, HML, UMD, RMW, and CMA denote the Fama–French–Carhart market, size, value-growth, momentum, profitability, and investment factors, respectively. R^2_{OS} denotes the out-of-sample coefficient of determination (in %). R^2_{COS1} denotes the (constrained) out-of-sample R^2 associated with a positive forecast restriction. R^2_{COS2} denotes the R^2 associated with a positive forecast restriction plus a positive slope restriction. The total sample is 1967:02–2016:12. The out-of-sample evaluation period starts in 1987:02 (Panels A-B) or 1997:02 (Panels C-D). In Panels B and D, the predictive regressions that generate the forecasts do not include the intercept.

	RM	SMB	HML	UMD	RMW	CMA
	Panel A: 1987:02–2016:12, intercept					
R^2_{OS}(%)	−4.73	−2.21	−3.02	2.16	−15.08	−0.95
R^2_{COS1}(%)	−5.02	−2.21	−3.02	0.25	2.37	−0.95
R^2_{COS2}(%)	−4.34	−2.14	−2.91	−0.76	1.14	−0.95
	Panel B: 1987:02–2016:12, no intercept					
R^2_{OS}(%)	−7.39	−3.66	−3.48	−5.61	−6.07	−0.37
R^2_{COS1}(%)	−6.78	−3.66	−3.48	−6.88	1.94	−0.37
R^2_{COS2}(%)	−6.78	−3.66	−3.48	−6.88	1.94	−0.37
	Panel C: 1997:02–2016:12, intercept					
R^2_{OS}(%)	0.32	−1.35	−4.27	2.27	−16.58	−0.68
R^2_{COS1}(%)	−0.19	−1.35	−4.27	0.15	2.45	−0.68
R^2_{COS2}(%)	0.33	−1.26	−4.12	−0.79	1.21	−0.68
	Panel D: 1997:02–2016:12, no intercept					
R^2_{OS}(%)	−2.30	−1.91	−5.36	−5.21	−6.03	0.19
R^2_{COS1}(%)	−1.76	−1.90	−5.36	−6.64	2.68	0.19
R^2_{COS2}(%)	−1.76	−1.90	−5.36	−6.64	2.68	0.19

Table A.9: Benchmark Trading Strategy: Alternative In-Sample Period

This table reports the performance evaluation measures associated with the benchmark trading strategy that is based on the out-of-sample forecasting power of the realized variance for the return of each factor. SMB, HML, UMD, RMW, and CMA denote the active strategies for the Fama–French–Carhart size, value-growth, momentum, profitability, and investment factors, respectively. P denotes the passive strategy. The statistics are the average return (Mean), standard deviation (SD), annualized raw Sharpe ratio (Sharpe), Skewness (Skew.), Kurtosis (Kurt.), and maximum drawdown (MDD). "Long" denotes the fraction of months in which the active strategy goes long in the factor. $\Delta U, \gamma = 3$, $\Delta U, \gamma = 5$, and $\Delta U, \gamma = 10$ represent the annualized change in average utility associated with a risk aversion coefficient of three, five, and ten, respectively. The total sample is 1967:02–2016:12 and the out-of-sample evaluation period starts in 1997:02.

	Mean(%)	SD(%)	Sharpe	Skew.	Kurt.	MDD(%)	Long(%)	$\Delta U(\%), \gamma = 3$	$\Delta U(\%), \gamma = 5$	$\Delta U(\%), \gamma = 10$
P	0.72	4.55	0.55							
SMB	1.05	7.64	0.47	0.12	4.86	−52.74	0.99	−2.89	−7.40	−18.67
HML	1.11	6.18	0.62	−0.67	5.22	−69.75	1.00	1.45	−0.64	−5.89
UMD	1.59	9.20	0.60	1.08	11.98	−63.99	0.88	−1.08	−8.75	−27.90
RMW	1.53	4.87	1.09	0.74	9.20	−29.80	0.91	9.19	8.83	7.94
CMA	1.17	4.64	0.88	−0.31	3.61	−51.32	1.00	5.26	5.16	4.92

Table A.10: Benchmark Trading Strategy: Alternative Weights

This table reports the performance evaluation measures associated with the benchmark trading strategy (with alternative weights) that is based on the out-of-sample forecasting power of the realized variance for the return of each factor. SMB, HML, UMD, RMW, and CMA denote the active strategies for the Fama–French–Carhart size, value-growth, momentum, profitability, and investment factors, respectively. P denotes the passive strategy. The statistics are the average return (Mean), standard deviation (SD), annualized raw Sharpe ratio (Sharpe), Skewness (Skew.), Kurtosis (Kurt.), and maximum drawdown (MDD). "Long" denotes the fraction of months in which the active strategy goes long in the factor. $\Delta U, \gamma = 3$, $\Delta U, \gamma = 5$, and $\Delta U, \gamma = 10$ represent the annualized change in average utility associated with a risk aversion coefficient of three, five, and ten, respectively. The total sample is 1967:02–2016:12 and the out-of-sample evaluation period starts in 1987:02.

	Mean(%)	SD(%)	Sharpe	Skew.	Kurt.	MDD(%)	Long(%)	$\Delta U(\%), \gamma = 3$	$\Delta U(\%), \gamma = 5$	$\Delta U(\%), \gamma = 10$
P	0.87	4.39	0.69							
SMB	1.08	8.49	0.44	−0.05	6.79	−60.36	0.99	−7.06	−13.39	−29.20
HML	1.37	6.60	0.72	−0.55	5.72	−75.11	1.00	1.59	−1.32	−8.60
UMD	2.19	10.37	0.73	0.91	13.87	−74.69	0.92	−0.08	−10.66	−37.11
RMW	1.84	5.53	1.15	0.71	10.97	−38.71	0.91	9.58	8.22	4.82
CMA	1.45	4.75	1.06	−0.37	4.17	−52.06	1.00	6.36	5.97	5.00

Table A.11: Benchmark Trading Strategy: Positive Slope Estimates

This table reports the performance evaluation measures associated with the benchmark trading strategy that is based on the out-of-sample forecasting power of the realized variance for the return of each factor. The slope estimates in the predictive regressions are truncated to non-negative values. SMB, HML, UMD, RMW, and CMA denote the active strategies for the Fama–French–Carhart size, value-growth, momentum, profitability, and investment factors, respectively. P denotes the passive strategy. The statistics are the average return (Mean), standard deviation (SD), annualized raw Sharpe ratio (Sharpe), Skewness (Skew.), Kurtosis (Kurt.), and maximum drawdown (MDD). "Long" denotes the fraction of months in which the active strategy goes long in the factor. $\Delta U, \gamma = 3$, $\Delta U, \gamma = 5$, and $\Delta U, \gamma = 10$ represent the annualized change in average utility associated with a risk aversion coefficient of three, five, and ten, respectively. The total sample is 1967:02–2016:12 and the out-of-sample evaluation period starts in 1987:02.

	Mean(%)	SD(%)	Sharpe	Skew.	Kurt.	MDD(%)	Long(%)	$\Delta U(\%), \gamma = 3$	$\Delta U(\%), \gamma = 5$	$\Delta U(\%), \gamma = 10$
P	0.87	4.39	0.69							
SMB	1.03	7.19	0.50	−0.30	6.23	−52.74	1.00	−3.96	−7.85	−17.58
HML	1.25	5.57	0.78	−0.72	5.85	−69.75	1.00	2.36	0.96	−2.56
UMD	1.65	7.54	0.76	−0.85	9.14	−72.51	1.00	2.55	−1.95	−13.20
RMW	1.39	4.64	1.03	−0.56	8.27	−41.73	1.00	5.73	5.46	4.78
CMA	1.31	4.31	1.05	−0.60	4.90	−51.32	1.00	5.34	5.42	5.64

Table A.12: Benchmark Trading Strategy: Bivariate Regressions

This table reports the performance evaluation measures associated with the benchmark trading strategy that is based on the out-of-sample forecasting power of both the realized variance for the return of each factor and the realized covariance with the market factor. SMB, HML, UMD, RMW, and CMA denote the active strategies for the Fama–French–Carhart size, value-growth, momentum, profitability, and investment factors, respectively. P denotes the passive strategy. The statistics are the average return (Mean), standard deviation (SD), annualized raw Sharpe ratio (Sharpe), Skewness (Skew.), Kurtosis (Kurt.), and maximum drawdown (MDD). "Long" denotes the fraction of months in which the active strategy goes long in the factor. $\Delta U, \gamma = 3$, $\Delta U, \gamma = 5$, and $\Delta U, \gamma = 10$ represent the annualized change in average utility associated with a risk aversion coefficient of three, five, and ten, respectively. The total sample is 1967:02–2016:12 and the out-of-sample evaluation period starts in 1987:02.

	Mean(%)	SD(%)	Sharpe	Skew.	Kurt.	MDD(%)	Long(%)	$\Delta U(\%), \gamma = 3$	$\Delta U(\%), \gamma = 5$	$\Delta U(\%), \gamma = 10$
P	0.87	4.39	0.69							
SMB	0.96	6.96	0.48	−0.46	6.53	−61.44	0.96	−4.18	−7.67	−16.42
HML	1.34	5.15	0.90	−0.51	4.79	−49.81	0.98	4.24	3.38	1.21
UMD	1.95	8.27	0.82	0.73	13.31	−63.99	0.92	4.03	−1.86	−16.58
RMW	1.53	5.10	1.04	0.01	8.38	−40.72	0.86	6.66	5.86	3.86
CMA	1.26	4.42	0.99	−0.78	5.68	−58.37	0.99	4.55	4.52	4.45

Table A.13: Alternative Trading Strategy

This table reports the performance evaluation measures associated with the alternative dynamic strategy, which is based on the out-of-sample forecasting power of the realized variance for the return of each factor. SMB, HML, UMD, RMW, and CMA denote the active strategies for the Fama–French–Carhart size, value-growth, momentum, profitability, and investment factors, respectively. The second row associated with each factor denotes the passive strategy, which is factor specific. The statistics are the average return (Mean), standard deviation (SD), annualized raw Sharpe ratio (Sharpe), Skewness (Skew.), Kurtosis (Kurt.), and maximum drawdown (MDD). "Long" denotes the fraction of months in which the active strategy goes long in the factor. $\Delta U, \gamma = 3$, $\Delta U, \gamma = 5$, and $\Delta U, \gamma = 10$ represent the annualized change in average utility associated with a risk aversion coefficient of three, five, and ten, respectively. The total sample is 1967:02–2016:12 and the out-of-sample evaluation period starts in 1987:02. In Panel B, the predictive regressions that generate the forecasts do not include the intercept.

	Mean(%)	SD(%)	Sharpe	Skew.	Kurt.	MDD(%)	Long(%)	$\Delta U(\%), \gamma = 3$	$\Delta U(\%), \gamma = 5$	$\Delta U(\%), \gamma = 10$
Panel A: intercept										
SMB	1.03	7.17	0.50	−0.30	6.29	−52.74	0.99	0.04	0.08	0.20
	1.03	7.19	0.50							
HML	1.25	5.57	0.78	−0.72	5.85	−69.75	1.00	0.00	0.00	0.00
	1.25	5.57	0.78							
UMD	1.76	6.48	0.94	−0.51	6.54	−46.69	0.92	3.95	5.73	10.19
	1.65	7.54	0.76							
RMW	1.49	4.18	1.24	−0.45	5.20	−28.64	0.91	2.03	2.52	3.75
	1.39	4.64	1.03							
CMA	1.31	4.31	1.05	−0.60	4.90	−51.32	1.00	0.00	0.00	0.00
	1.31	4.31	1.05							
Panel B: no intercept										
SMB	0.97	7.08	0.48	−0.41	6.22	−60.55	1.00	−0.39	−0.19	0.31
	1.03	7.19	0.50							
HML	1.25	5.57	0.78	−0.72	5.85	−69.75	1.00	0.00	0.00	0.00
	1.25	5.57	0.78							
UMD	1.58	7.21	0.76	−0.90	10.59	−66.25	0.74	0.07	0.64	2.07
	1.65	7.54	0.76							
RMW	1.31	4.00	1.13	−0.74	6.43	−29.91	0.65	0.10	0.76	2.43
	1.39	4.64	1.03							
CMA	1.31	4.31	1.05	−0.60	4.90	−51.32	1.00	0.00	0.00	0.00
	1.31	4.31	1.05							

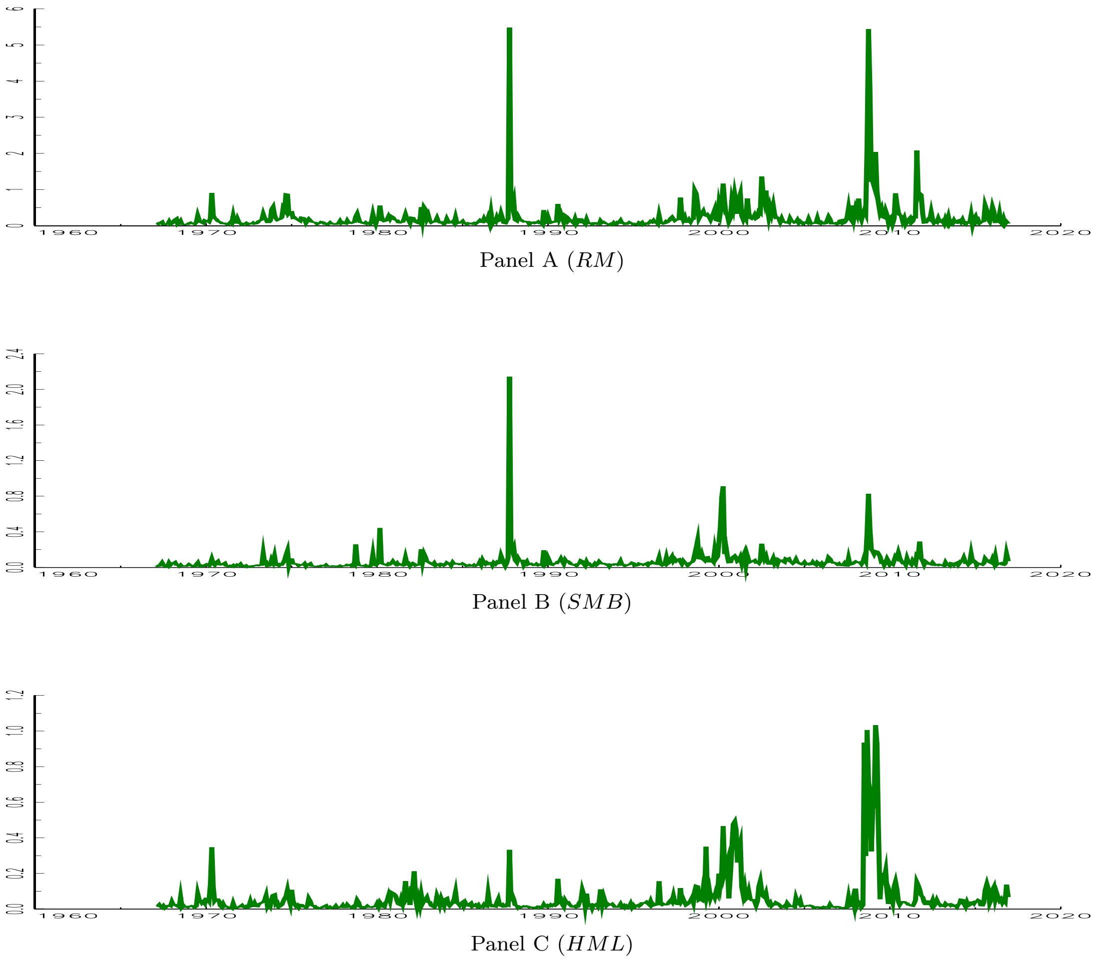


Figure A.1: Realized Variances: RM, SMB, and HML

This figure plots the realized variances (in %) of each equity factor. RM, SMB, and HML denote the Fama–French market, size, and value-growth factors, respectively. The sample is 1967:02–2016:12.

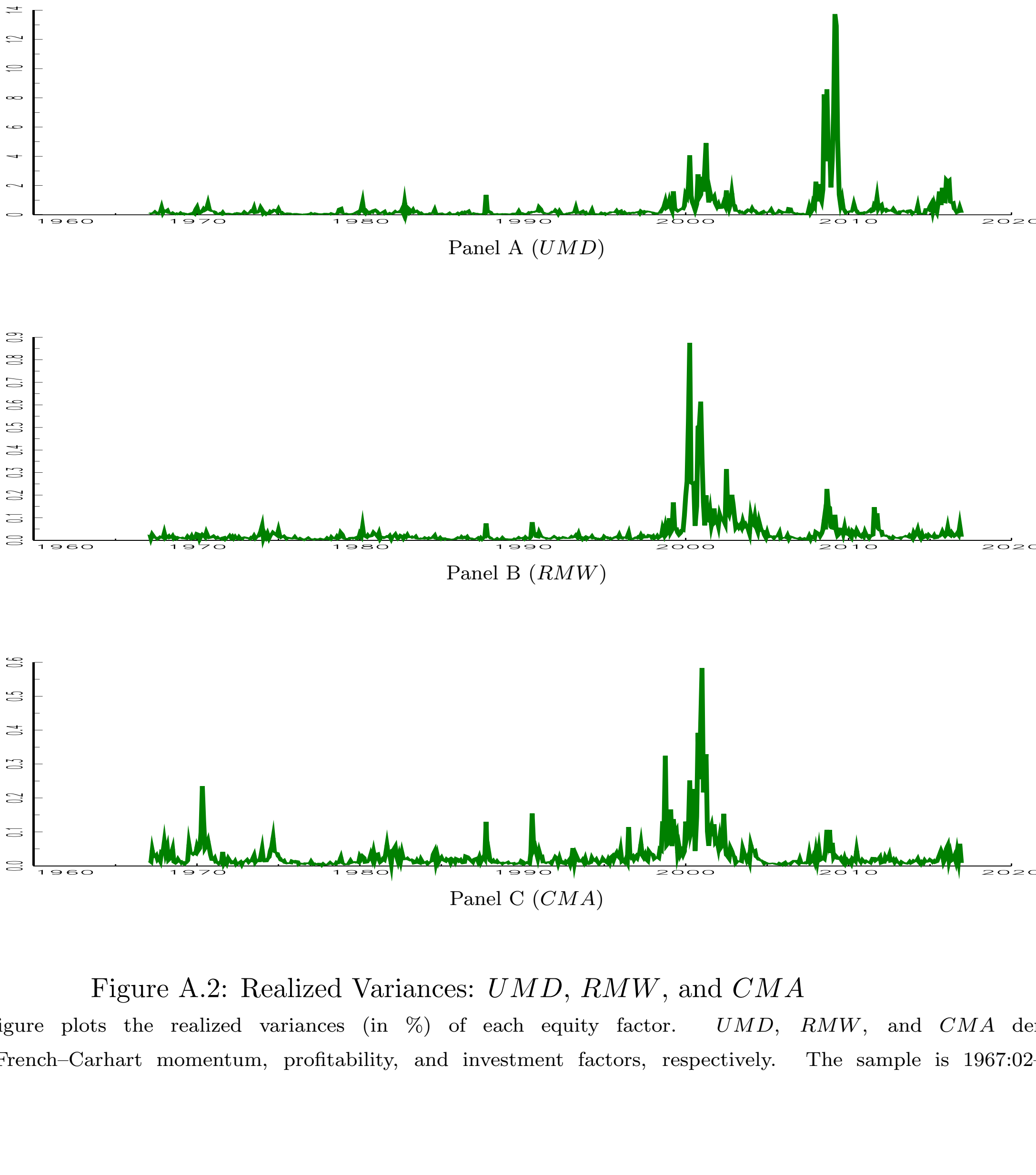
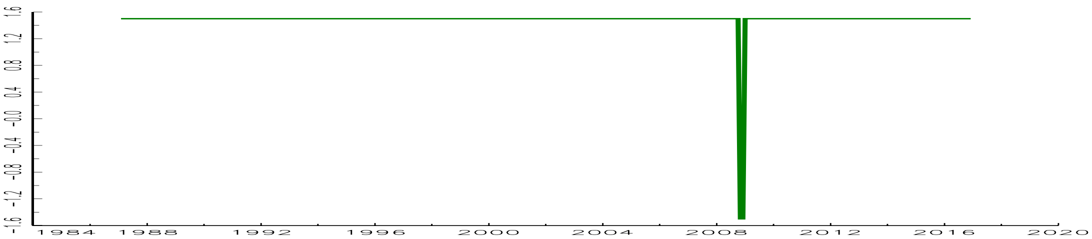


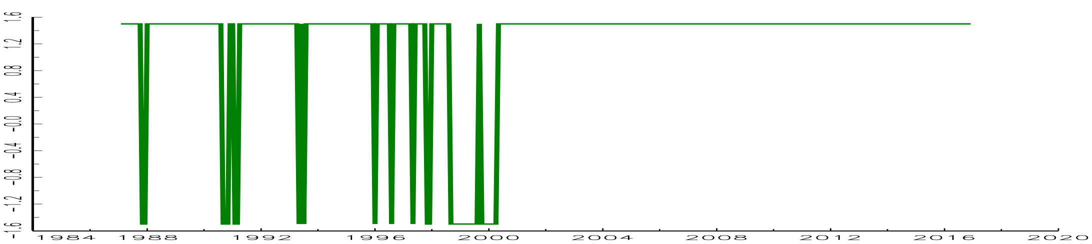
Figure A.2: Realized Variances: UMD, RMW, and CMA

This figure plots the realized variances (in %) of each equity factor. UMD, RMW, and CMA denote the Fama–French–Carhart momentum, profitability, and investment factors, respectively. The sample is 1967:02–2016:12.



Panel A (SMB)

Panel B (UMD)



Panel C (RMW)

Figure A.3: Factor Weights

This figure plots the weights in the factors associated with the benchmark dynamic trading strategy. SMB, UMD, and RMW denote the Fama–French–Carhart size, momentum, and profitability factors, respectively. The sample is 1987:02–2016:12.